Search Intermediaries, Subscriptions, and Advertising

Martino Banchio*

June 26, 2026

Abstract

Some search intermediaries rely on advertising revenue, monetizing the market power that search frictions confer on firms. Others charge consumers a participation (subscription) fee, monetizing the search improvement they provide to consumers. I endogenize the choice of monetization channel of a platform by embedding it into a sequential search model with differentiated products. The platform reduces search frictions for its users and collects revenue either via sponsored search or by charging subscription fees. As on-platform search becomes less frictional, monetizing shifts from advertising to subscription. Competition between search intermediaries may not be sufficient to stimulate investment in reducing frictions.

JEL Classification: D43, D83, L13.
Keywords: search intermediaries, platform monetization, advertising, subscriptions, sequential search, two-sided markets.

*Università Bocconi, IGIER, and Google Research. Email: martino.banchio@unibocconi.it. I am grateful to Christoph Carnehl, Nenad Kos, Suraj Malladi, Andrzej Skrzypacz, and the Market Algorithms team at Google Research for helpful comments and discussions. All errors are my own.

1 Introduction

Search intermediaries are platforms that help consumers find products. A traveler might consult TripAdvisor or Yelp when looking for a restaurant, or a real estate buyer may turn to Zillow or Redfin to find a good candidate home. These platforms' technology reduces the cost of inspecting options. As product variety grows, so does the value of search intermediation services.[1] While many digital platforms mediate similar offline search markets, their revenue models vary widely. In 2023, Yelp generated 1.34 billion dollars in revenue, with proceeds coming from paid advertising representing 96% of this figure. DoorDash, which intermediates a similar offline market, posted over 12 billion dollars in revenue the same year, advertising representing barely 8% and the vast majority relying instead on consumer-facing fees. Beyond the restaurant industry, as of 2025, an estimated 55 to 60% of OpenAI's revenue originates from paid subscriptions, while Zillow's advertising revenue consists of 65% of the total.

This paper develops a model to determine whether a search intermediary monetizes primarily through advertising or through user fees. I do so by endogenizing the revenue composition of a search intermediary, and I compare revenue from one-time user fees (subscriptions) and pay-for-prominence, or advertising. I show that the dominant revenue channel is determined by the platform's search *technology*, more specifically the technological advantage the platform offers over the outside option of unmediated search. Platforms with moderate technological advantage source most of their revenue from advertising, while revenue of platforms with superior technology originates from subscriptions.

Formally, I embed a stylized platform design problem into the canonical sequential search model of Wolinsky (1986). A finite number of symmetric firms sell differentiated products to a unit mass of consumers. Match values are idiosyncratic and unobservable prior to inspection, as are prices. In the baseline market, consumers engage in costly sequential search, sampling firms at random and learning match values and prices upon each visit. I study a platform endowed with a technology that reduces the per-search cost, under one of two monetization

[1] Brynjolfsson et al. (2025) collects data on the increase in ISBNs over a 14 year period. Using a broader definition of intermediation, Spulber (1996) estimates it to consist of a quarter of the US economy.

regimes. Under *subscriptions*, the platform charges consumers an ex-ante access fee and allows unlimited organic search. Under *advertising*, the platform offers free access but directs each consumer's first search to the firm that wins a second-price auction for the sponsored slot, while subsequent searches proceed organically.

I characterize equilibrium revenue under each regime in closed form. Subscription revenue equals the increment in the consumer's reservation match value from cheaper search, the excludable component of the surplus generated by the platform. Advertising revenue equals the value of the local monopoly position that the sponsored slot confers, which the auction fully extracts through competition in the advertising market. Importantly, the choice of monetization does not affect total surplus, but only its distribution among consumers, firms, and platform. This last observation allows for particularly transparent economic analysis.

Consider a platform that monetizes through advertising. Its revenue derives from the market power that search frictions confer on firms. The platform sells *prominence*, that is, the privilege of being the consumer's first point of contact, and the prominent firm profits from the local monopoly that arises because the consumer's next-best alternative requires costly further search, a mechanism reminiscent of the Diamond (1971) paradox. Note that, similarly because of the Diamond (1971) paradox at work, advertising does not increase prices in this model. If the platform's technology substantially reduces search frictions, it erodes the very market power on which advertising revenue depends. Instead, a platform monetizing through subscriptions derives revenue from the surplus that consumers receive when search becomes cheaper. As the platform's technology improves, consumers discover better matches more efficiently, and their willingness to pay for access grows monotonically. Eventually, as search technology improves the consumers' value for cheaper search exceeds the producers' willingness to pay for a captive audience, and subscriptions become the superior monetization scheme.

The model is highly stylized. As such, it captures some of the relevant features of many markets where a platform mediates costly search between buyers and sellers, but I make no attempt to closely describe any specific market. As examples, the introduction mentioned food delivery apps such as Doordash, Glovo, and various competitors, who offer consumers a simple, recognizable interface to browse the menu offering of local restaurants. This technology reduces the search frictions that restaurant directories like Yelp monetized through advertising.[2] In turn, this has led to a larger share of revenue to come from consumers. Similarly, travel aggregators and online travel agencies catalog options for consumers, aggregating product offerings to simplify the search task for travelers. At times, these services have monetized differently, charging both consumers and firms. This model's results may help understanding the dynamic of the industry as function of its technology.

Perhaps the most recent example comes from web search. Traditional search engines offer meaningful reductions in search frictions, yet their very interface requires users to click through links, read pages, and compare options. These platforms almost universally adopted advertising. Emerging AI-powered search assistants instead synthesize information and arguably reduce the cognitive load required by the user for search. Consistent with the model's prediction, these platforms have primarily adopted subscription pricing.[3] As AI-powered tools increasingly mediate economic activity across many domains and reduce search frictions more broadly, the question of how search intermediaries should be monetized takes on first-order importance.

I conclude the paper with a discussion of how competition among platforms affects the results. A worse intermediary monetizes higher firm market power, and can in turn rebate ad revenue to consumers to subsidize adoption. The market may tip toward the technologically lagging intermediary for some values of the primitives.

Relation to the literature. This paper follows the tradition of the sequential consumer search literature initiated by Diamond (1971), McCall (1970), and Wolinsky (1986). While Diamond's paradox introduced the idea that arbitrarily small

[2]Most food delivery apps separate the fees for delivery from intermediation fees which are often charged to the restaurants and passed on to consumers even if the consumer chooses the pickup option. See for example "Each listed item [Ed: for pickup on Doordash] costs around ten to 20 percent more than its original price in-situ", from `https://www.cornellsun.com/article/2025/04/the-hidden-price-of-convenience-is-doordash-worth-it`. I show that in the model these fees are exactly equivalent to the fixed customer fee I call subscription.

[3]As of 2025, the major AI assistant providers charge subscription fees for their most capable models, while several experiment with advertising integration, suggesting the industry has not fully settled. Other forces, including high inference costs and the absence of mature ad-targeting infrastructure, may also contribute to the prevalence of subscriptions. The model highlights an equilibrium channel that operates independently of these supply-side considerations.

but positive search costs may eliminate price competition, the differentiated product model of Wolinsky (1986) has provided a framework for studying the effect of consumer search on prices and competition in the product market. This paper too extends the Wolinsky (1986) model by introducing a platform that reduces search costs for its users, and computing the platform's revenue under different monetization strategies. To do so I also leverage the definition of "prominent" firm of Armstrong, Vickers, and Zhou (2009).

The paper also relates to the two-sided markets literature (Rochet and Tirole, 2003; Armstrong, 2006; Rochet and Tirole, 2006). A central insight of that literature is that platforms may optimally subsidize one side of the market to extract surplus from the other. In this model instead the revenue split is not driven by cross-group externalities, but instead by the platform's technology. Unlike Gehrig (1993) and Rubinstein and Wolinsky (1987), who model a search intermediary as a monopolist or oligopolist who captures a bid-ask spread, in this model the intermediary simply reduces search frictions. All consumers share the same search cost, unlike in Salz (2022).

The literature on search advertising, starting with Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007), analyzed the auction mechanisms used by search platforms. In particular, Athey and Ellison (2011) studies details of the advertising auctions in the context of consumer search. Closer to this model, several papers embed platforms into Wolinsky-style search models to study ad pricing and paid placement on search intermediaries. Among them, Eliaz and Spiegler (2011) introduce a search engine that controls the search pool and monetizes via a price-per-click mechanism, and Chen and He (2011) study how paid placement alters consumer search behavior and firm pricing. Unlike previous work, this model abstracts away from the specifics of the auction pricing and selection rules to compare the fundamental business models available to search intermediaries, and in particular whether such platforms should engage in the advertising business at all.

Finally, this paper relates to the growing literature on the economics of AI and digital platforms. As AI-powered tools increasingly mediate economic transactions, the question of how these intermediaries should be monetized—and the incentive distortions that different financing models create—takes on first-order importance. Unlike much of the recent literature, which focuses on mechanism design for selling AI-generated content (for example, Duetting et al. (2024); Dubey et al. (2024); Soumalias, Curry, and Seuken (2024); Bergemann, Bonatti, and Smolin (2026)) or on advertising within AI-assisted interactions (Banchio, Mehta, and Perlroth (2025); Bhawalkar, Psomas, and Wang (2025); Banchio, Liu, and Perlroth (2026)), this paper proposes a positive theory of platform monetization that applies across the range of settings in which platforms mediate consumer search.

Organization. The remainder of the paper is organized as follows. Section 2 presents the model of the product market and characterizes the baseline equilibrium without a platform. Section 3 introduces the platform, derives equilibrium under each monetization regime, establishes the main threshold result, and characterizes welfare. Section 4 discusses how competition may shape the platforms' monetization models and their incentives to innovate. I conclude with a discussion of the results and their limitations in Section 5. Proofs not contained in the main text are collected in the Appendix.

2 The Product Market

Consider a market populated by a unit mass of risk-neutral consumers and $N \geq 2$ identical firms, indexed by $j \in \{1, 2, \ldots, N\}$. Each consumer has unit demand for a single, indivisible good. Firms produce at constant marginal cost, normalized to zero. If a consumer purchases from firm j at price p_j, her utility is

$$u_j = v_j - p_j,$$

where v_j is a match value reflecting the consumer's idiosyncratic taste for firm j's product. Match values are drawn independently across all consumer–firm pairs from an absolutely continuous distribution F with density f, supported on $[0, V]$. A consumer who does not purchase from any firm receives a utility of zero. I assume the following regularity condition.

Assumption 1. The distribution F is twice continuously differentiable on its domain, and its density $f(x)$ is strictly log-concave.

This assumption ensures uniqueness of the symmetric pricing equilibrium and quasi-concavity of the firm's profit function.[4]

Further, I assume that search frictions are sufficiently small so that consumers benefit from search and the market is non-trivial.

Assumption 2. The search cost satisfies $0 < s < \hat{s}$, where $\hat{s} \equiv \int_{x_0}^{V} (v - x_0)\, dF(v)$ and x_0 is the unique solution to $x_0 f(x_0) = 1 - F(x_0)$.

By Assumption 1 the hazard ratio is increasing, so x_0 exists and is unique.

Information and search. Prior to engaging in search, consumers observe neither match values nor prices. To learn the pair (v_j, p_j) for any firm j, a consumer must inspect that firm at a cost of $s > 0$. Search is sequential: at each stage, the consumer draws one firm uniformly at random from the set of firms not yet inspected, observes its price and her match value, and decides whether to stop and purchase from one of the firms already inspected, or to continue searching. Consumers have perfect, costless recall.

Let the index of search be $t \in \{1, \dots, N\}$. Each consumer carries a history at stage t, that is, a sequence $h^t = \left(j_k, p_{j_k}, v_{j_k}\right)_{k=1}^{t}$ recording the identity, price, and realized match value of each firm inspected. We write $\mathcal{H}^t$ for the set of all histories of length t and $\mathcal{H} = \bigcup_{t=1}^{N} \mathcal{H}^t$ for the set of all possible histories.

A *strategy* for the consumer consists of a stopping rule $\tau \colon \mathcal{H} \to \{\text{stop}, \text{continue}\}$ and a purchase rule $\gamma \colon \{h \in H | \tau(h) = \text{stop}\} \to \{1, \dots, N\} \cup \emptyset$ mapping each (terminal) history to one of the inspected firms (or to no purchase). A *strategy* for firm j is a price $p_j \in \mathbb{R}_+$.

2.1 Equilibrium

I study symmetric equilibria in which all firms charge a common price.

Definition 1. A *symmetric Bayesian Nash equilibrium* consists of a price $p_N^* \in \mathbb{R}_+$, consumer beliefs $p^e \in \mathbb{R}_+$, and a consumer strategy (τ, γ) such that:

(i) **Consumer optimality.** Given beliefs p^e, each consumer's stopping and purchase rules maximize her expected utility.

[4]This assumption is satisfied by the uniform, normal, and logistic distribution, among others.

(ii) **Firm optimality.** Given the consumer's strategy, no firm can strictly increase its expected profit by a unilateral deviation from p_N^*.

(iii) **Consistency.** Beliefs are correct on the equilibrium path: $p^e = p_N^*$.

I restrict attention to *passive beliefs*: upon observing an off-path price $\hat{p} \neq p_N^*$ at some firm, a consumer does not revise her belief that the remaining unsampled firms charge p_N^*.

Wolinsky (1986) characterizes the equilibrium of this model, which I use as a baseline. I summarize the key elements here.

Consumer's strategy. Suppose all firms charge the equilibrium price p_N^* and consumers hold the correct belief $p^e = p_N^*$. Under passive beliefs, the consumer's continuation value depends only on the best match value discovered so far, making the problem stationary for all searches $t < N$. The consumer continues searching if and only if her best net "discovered" utility $v - p_N^*$ falls below a threshold which captures the option value of future search. This reservation value x^* is the unique solution to

$$\int_{x^*}^{V} (v - x^*)\, dF(v) = s. \tag{1}$$

Assumption 2 ensures that $x^* \in (0, V)$; uniqueness is guaranteed by the strict monotonicity of the left-hand side. The reservation value depends on the search cost s but not on N nor on the symmetric equilibrium price.

Firm's Strategy. Each firm sets its price to maximize expected profit, taking into account that she will be drawn at a uniformly random position in the consumer's search sequence. In a symmetric equilibrium, all firms share the market equally. The equilibrium price is then determined by the first-order condition of the firm's profit function, evaluated at the symmetric price.

Proposition 1 (Wolinsky, 1986)**.** *Under Assumptions 1 and 2, the equilibrium price is*

$$p_N^* = \left(1 - F(p_N^*)^N\right)\left[\frac{1 - [F(x^*)]^N}{1 - F(x^*)} f(x^*) \;-\; N \int_{p_N^*}^{x^*} [F(v)]^{N-1} f'(v)\, dv\right]^{-1}, \tag{2}$$

where x^* is defined by Equation (1).

The consumer's choice together with the firm's pricing above characterize the unique symmetric equilibrium. Uniqueness follows from two observations. First, the reservation value x^* is the unique solution to (1), since the left-hand side is strictly decreasing (Lemma 3 in the appendix). Second, given x^*, the symmetric price is uniquely determined by the first-order condition, because the firm's profit function is strictly quasi-concave under Assumption 1 (see the appendix of Anderson and Renault (1999)).

In particular, the equilibrium simplifies considerably for large N. Since $F(x^*) < 1$, the terms $[F(x^*)]^N$ and $[F(v)]^{N-1}$ vanish as $N \to \infty$.

Corollary 1. *As $N \to \infty$, the equilibrium price converges to*

$$p^* \equiv \lim_{N\to\infty} p_N^* = \frac{1 - F(x^*)}{f(x^*)} = \frac{1}{\rho(x^*)},$$

where $\rho(x^*) = f(x^*)/[1-F(x^*)]$ is the hazard rate of F evaluated at the reservation value.

The equilibrium pricing formula is similar to the monopoly pricing formula, only it is evaluated in the threshold match value x^*. Intuitively, if consumers were to become more selective (that is, if x^* were to rise) the hazard rate would increase, competition would intensify, and prices would fall. In equilibrium, consumers balance selection against their search costs.

Before starting search, the consumer's utility is the option value of the search problem. In the large-market limit, this takes the simple form

$$U_{\text{off}} = x^* - p^*.$$

Since $x^* > 0$ and $p^* < x^*$ (which follows from Assumption 2), consumer welfare is strictly positive.

3 Search Intermediary and its Monetization

A platform (search intermediary) $\mathcal{P}$ enters the product market endowed with a search technology that reduces the per-search friction from s to $0 \leq s_p < s$. Firms can not distinguish whether a consumer discovers their product through the platform or on the offline market, hence their equilibrium price reflects their inference about consumers' choices. I analyze two pure monetization regimes, subscriptions and advertising, and derive the platform's optimal revenue under each. I then compare the two to establish the paper's main result.[5] For ease of exposition I work in the large-market limit, but all results extend to the finite-market case, at the expense of additional algebra.

First, suppose the platform offers its services for free. In this case all consumers search through the platform: they enjoy lower search costs and experience no downsides by doing so. Because the left-hand side of Equation (1) is strictly decreasing in x^*, the threshold x^* is a strictly decreasing function of the search friction s. Hence, $x_p^* > x^*$, where the threshold x_p^* solves $\int_{x_p^*}^{V}(v - x_p^*)\,dF(v) = s_p$. Intuitively, lower search costs make consumers more selective. Conversely, the equilibrium price p^* is a strictly increasing function of s, because lower search frictions decrease the customers' captivity, thus reducing market power in the product market. Firms, anticipating that all consumers will search through the platform, set an equilibrium price $p_p^* = 1/\rho(x_p^*) < p^*$. Finally, consumer welfare is a strictly decreasing function of the search friction, so the platform delivers utility $U_p = x_p^* - p_p^* > U_{\text{off}}$ to consumers by intensifying competition and improving matching. The platform increases total welfare in the market, so a natural question is what fraction of this welfare can the platform appropriate via monetization.

3.1 Subscriptions

Charging subscriptions is an attempt to capitalize on consumer surplus. The platform may charge each consumer a fixed fee $K \geq 0$ for access. Adopters search at cost s_p while non-adopters search offline at cost s. The equilibrium price depends on the fraction of adopters, but I assume that firms cannot discriminate based on the acquisition channel.[6]

[5] Section 3.3 also discusses the mixed monetization model.

[6] This is a simplifying assumption that does not affect the qualitative insights of the paper. In fact, it may well be the case that consumers that reach the firm through certain channels observe different prices. Some online travel agencies actively require participants to offer their lowest price through their platform, while restaurants often offer higher menu prices on food delivery platforms to offset costs. In the context of the model, I assume price discrimination away to avoid clouding the main economic takeaways.

Proposition 2 (Subscription Revenue)**.** *Under the subscription regime, the optimal fee is*

$$K^* = x_p^* - x^*.$$

In equilibrium all consumers adopt the platform, firms charge p_p^**, and total platform revenue is* $\Pi_{\text{sub}} = K^*$*.*

Proof. If all consumers adopt, previous analysis established that firms find optimal to charge price p_p^*. Consumers receive utility $U_p - K^* = (x_p^* - p_p^*) - (x_p^* - x^*) = x^* - p_p^*$ by adopting the search engine. A consumer who deviates by refusing to join the platform must search offline at cost s, yielding reservation value x^*, but she faces the lower equilibrium price p_p^* induced by the platform's other users. Hence a deviating consumer receives the same utility $x^* - p_p^*$, and does not strictly gain from deviation. For any $K < K^*$, adoption is a strictly dominant strategy for consumers. Taking $K \to K^*$ shows that the supremum of the platform's revenue is exactly K^*. □

The platform capitalizes on the improvement it induces in the quality of consumer-firm matching. Instead, the equilibrium price reduction is non-excludable, because all consumers benefit from lower prices regardless of whether they use the platform. Hence, subscription can charge only for the excludable value provided to consumers, the improved matching that comes from cheaper search. Consumers retain the price reduction, so $U_{\text{sub}} = x^* - p_p^* > x^* - p^* = U_{\text{off}}$, so they are strictly better off than in the offline market.

3.2 Advertising

I model advertising as an attempt to capitalize on producer surplus. In this model of advertising, the platform offers free access ($K = 0$) and sells the first search slot via a second-price auction among the N firms. Thus, the platform is selling *prominence* in the sense of Armstrong, Vickers, and Zhou (2009). The timing is as follows. First, all N firms simultaneously submit bids for the slot, the highest bidder wins, and they pay the second-highest bid, with ties broken uniformly. Second, all firms simultaneously set prices, observing the outcome of the auction.

In particular, the winning firm may condition its price on having won.[7] Third, consumers search. Each consumer's first search, if mediated by the platform, is directed to the winning firm, and subsequent searches proceed organically among the remaining $N-1$ firms in random order. All searches through the platform cost s_p, and since the platform is free all consumers use it. Consumers are aware that their first search result is sponsored, but their purchase decision depends only on the realized match value and price.

A consumer who arrives at the sponsored firm accepts if and only if $v-p \geq U_p = x_p^* - p_p^*$, i.e., $v \geq U_p + p$. The advertising firm maximizes $\pi(p) = p[1-F(U_p+p)]$.

Lemma 1. *In the large market limit, the sponsored firm charges the organic market price:* $p_{\text{ad}}^* = p_p^*$.

Proof. Substituting $p = p_p^*$ into the first-order condition gives $p_p^* = 1/\rho(x_p^* - p_p^* + p_p^*) = 1/\rho(x_p^*)$, which holds by definition of p_p^*. Uniqueness follows because $\log\pi(p) = \log p + \log[1-F(U_p+p)]$ is strictly concave: the first term is concave and the second is strictly concave by Assumption 1. □

While it could do so, the sponsored firm does not charge a premium over the organic price. The slot's value derives entirely from the platform's allocation of consumers' attention, instead of pricing power.[8] The advertising firm is inspected first by every consumer, so its revenue is $p_p^*(1-F(x_p^*))$. In the symmetric auction, firms bid their value for the slot, and the platform extracts this value as revenue.

Proposition 3 (Advertising Revenue)**.** *Under Assumptions 1 and 2, the advertising revenue from a second-price auction is* $\Pi_{\text{ad}} = p_p^*\left[1-F(x_p^*)\right] = \frac{[1-F(x_p^*)]^2}{f(x_p^*)}$.

Proof. Since firms are symmetric, each has the same value w for the slot and this is a complete-information auction, hence the platform earns w.[9] The formula follows from substituting $p_p^* = 1/\rho(x_p^*)$ into $p_p^*[1-F(x_p^*)]$. □

[7] This assumption can be relaxed without qualitatively affecting the results.

[8] The symmetric pricing result $p_{\text{ad}}^* = p_p^*$ is specific to the large-market limit. In finite markets, the prominent firm and non-prominent firms charge different prices, as characterized by Armstrong et al. (2009). I discuss the finite-N case in Section 5.

[9] Because all firms expect the same benefit from prominence, the auction is degenerate and any firm can win at price w. Platform revenue is the same under any standard auction format.

Remark 1 (No price distortions). Advertising introduces no price distortion: consumers face p_p^* whether encountering a firm organically or through an ad. Any distortion is purely allocative (the consumer's first search is directed rather than random), but since firms are ex-ante symmetric, this is welfare-neutral in the baseline model.[10]

3.3 Revenue Composition

The revenue from advertising and subscriptions can be written solely in terms of the platform's search cost:

$$\Pi_{\text{sub}}(s_p) = x_p^*(s_p) - x^*,$$

$$\Pi_{\text{ad}}(s_p) = \frac{\left[1 - F\left(x_p^*(s_p)\right)\right]^2}{f\left(x_p^*(s_p)\right)},$$

where $x_p^*(s_p)$ is defined implicitly by (1). To build intuition, consider the two extremes.

When the platform offers only a marginal improvement over offline search ($s_p \approx s$), subscription revenue is close to zero. To see this, note that the threshold x^* is increasing as a function of the search friction, and $\lim_{s_p \to s} x_p^*(s_p) = x^*$. Such a platform does not deliver much incremental value to the consumers, who in turn are not willing to pay much for its service. Advertising revenue, however, does not depend on the value delivered to consumers. Instead, it relies on the *market power* enjoyed by the firms. A platform that barely reduces search costs can sell valuable advertising slots, because search frictions endow firms with pricing power and create a captive audience, that is, $p^* > 0$ and $1 - F(x^*) > 0$. The platform's advertising relies on the existing market imperfection, delivering revenue $\Pi_{\text{ad}}(s_p) \approx p^*(1 - F(x^*)) > 0$.

Consider now instead a truly transformative search technology ($s_p \approx 0$). Consumers become highly selective, and competition among firms drives prices toward marginal cost. Firms can neither charge a meaningful price nor expect a

[10] If the model incorporated firm heterogeneity, the auction would allocate the slot to the firm with the highest willingness to pay, which need not maximize consumer surplus. I discuss targeting in Section 5.

high probability of sale. Advertising revenue, as the product of price and purchase probability, converges to zero. Subscription revenue, by contrast, captures the match quality improvement $x_p^* - x^* \approx V - x^*$: the consumer is willing to pay for the platform's ability to find her a near-perfect match, something she cannot replicate through costly offline search.

Remark 2. Interestingly, this model delivers as a side product an insight discussed in the literature by Athey and Ellison (2011) and White (2013). Search intermediaries that profit from advertising may lack incentives to improve their search technology, because advertising profit relies on the pricing power of the firms, which in turn depends on search frictions. Reducing frictions directly undermines the market power of advertisers and reduces revenue. By contrast, a subscription-funded platform captures value that grows with its technology's capability.

The following result extends the intuition above through single-crossing.

Theorem 1. *Under Assumptions 1 and 2, there exists a unique threshold $\bar{s} \in (0, s)$ such that $\Pi_{\text{ad}}(\bar{s}) = \Pi_{\text{sub}}(\bar{s})$. For more advanced search technologies $s_p < \bar{s}$, subscriptions strictly dominate advertising. For less advanced technologies $s_p > \bar{s}$, advertising strictly dominates subscriptions.*

Proof. Let $\Delta(s_p) \equiv \Pi_{\text{sub}}(s_p) - \Pi_{\text{ad}}(s_p)$. The previous analysis proves that $\lim_{s_p \to 0} \Delta(s_p) > 0$ and $\lim_{s_p \to s} \Delta(s_p) < 0$. Lemma 7 in Appendix A shows that Δ is continuous in s_p, so the intermediate value theorem guarantees existence of at least one $\bar{s} \in (0, s)$ with $\Delta(\bar{s}) = 0$.

To establish uniqueness it suffices to show that Δ is strictly monotone. It is a simple exercise I carry out in Appendix A to show that subscription revenue is strictly decreasing and advertising revenue is strictly increasing in s_p, which allows me to conclude. □

The threshold $\bar{s}$ is implicitly defined by

$$x_p^*(\bar{s}) - x^* = p_p^*(\bar{s})\Big[1 - F(x_p^*(\bar{s}))\Big].$$

The left-hand side is the incremental match quality that the platform delivers, or in other words the gap between what the user expects to find on vs off the platform. Instead, the right-hand side is the expected revenue per consumer that a firm earns when granted a captive first inspection, a quantity that depends on both pricing power and the probability that the consumer purchases. At the threshold, the consumer's willingness to pay for better matching equals the firm's willingness to pay for a captive audience.

Discussion. This characterization offers an interpretation of recent trends in the search industry. Traditional search engines entered a market in which consumers searched by visiting individual websites, comparing options manually, and navigating link directories. These platforms offered a significant improvement, but the fundamental interaction remained sequential: the consumer still had to click through links, read pages, and evaluate options herself. In the language of the model, these platforms reduced s but remained above $\bar{s}$. The residual frictions left firms with meaningful pricing power, which sustained large advertising revenues. AI-powered search assistants represent a qualitatively different technology. By synthesizing information and delivering direct answers, these tools compress what previously required multiple searches into a single interaction, pushing s_p closer to zero and into the subscription-dominant region. The early pricing strategies of AI assistants, primarily subscription-based with limited advertising experiments, are broadly consistent with this prediction, though many other factors (including inference costs and the absence of mature ad-targeting infrastructure) likely contribute to the observed pattern.

It is perhaps important to note that many platforms monetize by means of both channels discussed above. Because advertising does not introduce price distortions, a platform that simultaneously charges a subscription fee K and runs an advertising auction faces the same equilibrium pricing, and simply enjoys both revenue streams. The paper can therefore be read as characterizing the platform's revenue *composition* as a function of its technological advantage.

Remark 3 (Comparative statics in offline search cost). The threshold $\bar{s}$ depends on the offline search cost s, which represents the consumer's outside option. Worse outside options increase firms' market power, in turn increasing advertising revenue at any given s_p. The composition threshold $\bar{s}$ shifts downward, so a more capable platform is needed for subscriptions to dominate. This suggests that the dominant monetization model may vary across product categories, with advertising more prevalent in markets where consumers face high intrinsic search costs.

3.4 Welfare

Finally, note that the choice of monetization regime affects distribution but not efficiency.

Proposition 4 (Welfare)**.** *Total surplus is x_p^* under any monetization regime. Consumers are better off under advertising, while firms are better off under subscriptions. Under either regime, consumers are strictly better off than in the offline market.*

The simple proof is in Appendix A. The result follows from the observation that advertising does not introduce price distortions, and that match quality is the only determinant of consumer surplus since there is no price dispersion.

Figure 1 illustrates the revenue comparison for a uniform match distribution. Total gains from trade without a platform are 0.5, and the figure shows that the platform increases total gains from trade monotonically with its search friction. As the platform's technology improves, producer surplus decreases, and ad revenue decreases as well. On the contrary, the fraction of surplus that accrues to consumers increases and so does subscription revenue. The total revenue of a platform monetizing through both the channels above is the sum of the green- and red-shaded areas.

4 Platform Competition

The baseline model considers a monopolist platform. In practice, search intermediaries with different technological capabilities compete for consumers. In this section, I extend the model to Bertrand-style duopolistic competition between asymmetric platforms. I characterize the revenue composition of the dominant platform, and I show that the market may tip toward the technologically lagging search intermediary.

Consider two platforms, $\mathcal{P}_1$ and $\mathcal{P}_2$, endowed with search technologies $s_1 < s_2$. Platform $\mathcal{P}_1$ offers superior technology, so consumers searching on $\mathcal{P}_1$ face lower search costs and thus adopt a higher reservation value, $x(s_1) = x_1 > x_2 = x(s_2)$. As before, all firms appear on both platforms, isolating competition to the consumer

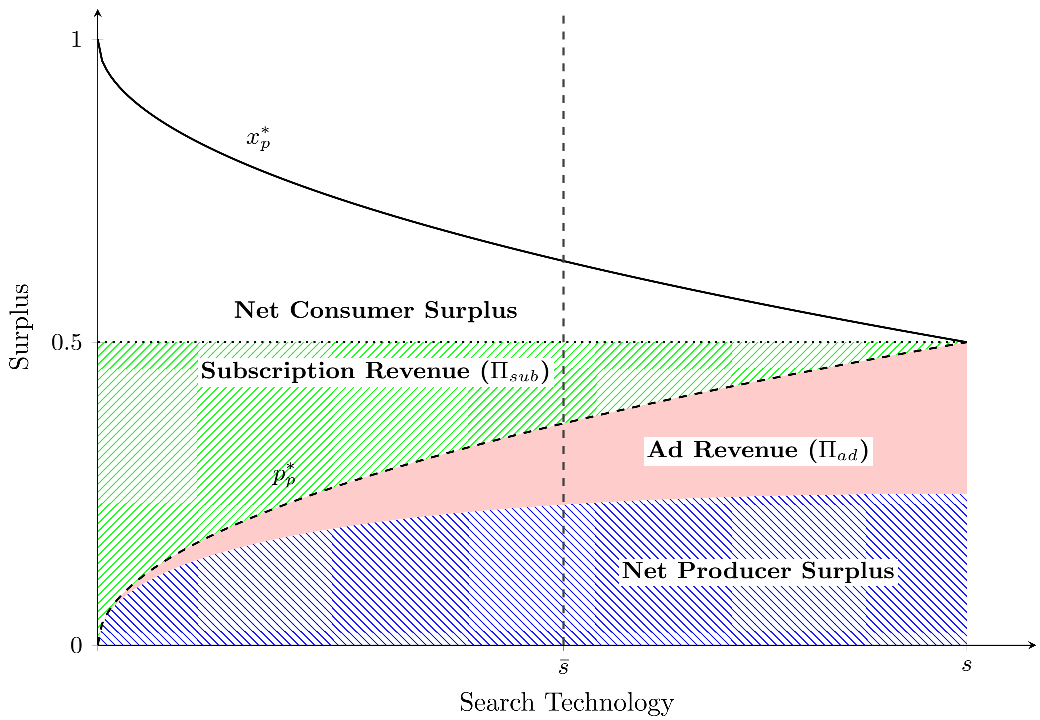

Figure 1: The figure depicts the portion of the total gains from trade captured by the platform for varying levels of its search technology, and its source, for $v \sim U[0,1]$ and $s = 0.125$.

side. Firms cannot discriminate between consumers arriving from different platforms, so in equilibrium they charge a common price p_M, which depends on which platform captures consumers, or the *dominant* platform.

Platforms compete à la Bertrand for consumers by setting an access fee $K_i \in \mathbb{R}$. In particular platforms may set $K_i < 0$, *subsidizing* consumers through costly ancillary services (e.g., cloud storage, email, or digital maps) provided for free. Since prices are common across platforms, each consumer chooses the search intermediary delivering the largest gross utility, $U_i = x_i - K_i$.

Platforms compete over consumers along the gross utility margin, but they do so through different channels. A high-quality platform ($s_1 \approx 0$) can deliver consumer utility through the high match value $x_1 \approx V$ it induces: lowering its subscription fee passes this matching surplus directly to the consumer. A low-quality platform ($s_2 \approx \bar{s}$) cannot substantially improve match quality, but it can monetize the market power of its advertisers. By running ads and rebating the advertising revenue to consumers through ancillary services, it can offer a subsidy $K_2 < 0$ equal in magnitude to its advertising take.

More generally, each platform can finance consumer subsidies from both channels. If platform i were to serve a consumer at a price p_M, it could gather $p_M(1 - F(x_i))$ in advertising revenue for that consumer. Hence, a platform serving the entire market may make a maximum feasible offer to consumers of

$$B(x_i) \equiv x_i + \frac{[1 - F(x_i)]^2}{f(x_i)},$$

which combines the matching value x_i it induces with the fully rebated advertising revenue given the market price when platform i is dominant.

The market tips entirely to the platform able to make the highest feasible offer. The resulting profit structure follows standard Bertrand logic. The dominant platform $\mathcal{P}_j$ with $B(x_j) > B(x_{-j})$ attracts all consumers by offering gross utility just above $B(x_{-j})$, leaving the rival unable to match. In equilibrium, the dominant platform sets its fee K_j so that $x_j - K_j = B(x_{-j})$, earning profit $B(x_j) - B(x_{-j})$ while the losing platform earns zero.

The function B fully determines the equilibrium outcome. Its first term x_i increases monotonically as the platform's technology improves, while its second term $[1 - F(x_i)]^2/f(x_i)$ decreases monotonically in x_i as better matching erodes advertisers' market power. The next lemma characterizes their compositional effect.

Lemma 2. *Under Assumption 1, the function $B(x)$ is strictly convex on $(0, V)$. It has an interior minimum at $x_{\min} \in (0, V)$.*

The proof of this result is in the appendix. The value of $B(x(s))$ can be visualized by adding up the net consumer surplus, the subscription revenue, and the ad revenue regions in Figure 1. The next result is an immediate consequence of the lemma above.

Proposition 5. *There exists a continuous function $g(s)$ such that platform 2 mediates all consumer search in equilibrium if and only if $s_2 > g(s_1)$. Otherwise, platform 1 mediates all consumer search in equilibrium.*

The economics are intuitive. A platform with technology $s_{\min}$ has improved search enough to erode the product firms' market power, reducing the revenue available from advertising. It has however not reduced search frictions enough to generate the large matching gains that would justify subscription-based competition. Its rival, by retaining large search frictions, intermediates a product market where firms enjoy substantial pricing power and in turn earns high advertising revenue, which it can rebate to consumers. The technologically inferior platform wins by redistributing the rents generated by the market imperfection.

The function g is decreasing for $s_1 < s_{\min}$ such that $x(s_{\min}) = x_{\min}$, while for $s \geq s_{\min}$ it is the identity function. Interestingly, this function outlines two qualitatively different competitive regimes, depicted in Figure 2.

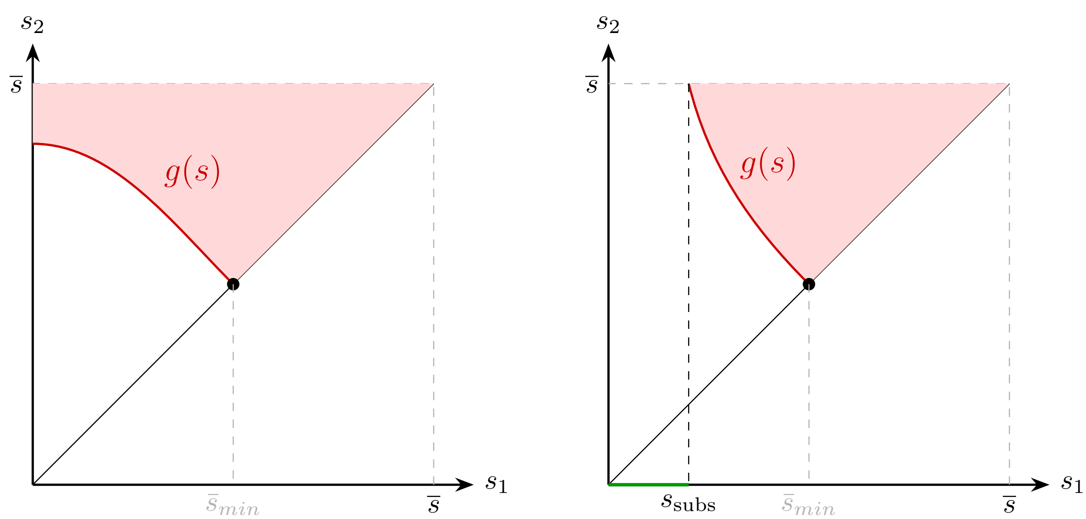

Figure 2: Shaded regions indicate technology pairs at which the inferior platform dominates.

In the left panel, the function g is strictly bounded above by the offline search technology $\bar{s}$. In this case, even a perfect search technology cannot deliver as much gross surplus as a legacy platform operating at or near the offline threshold. In this regime, a sufficiently low quality incumbent can always dominate the market, irrespective of how advanced the entrant's technology is. The market is entrenched, in the sense that the incumbent's ability to fund consumer subsidies through advertising insulates it from technological change. Innovation incentives are weak, because a platform cannot profitably attract consumers by improving its search technology.

Instead in the right panel a sufficiently advanced technology can overcome the advertising-funded incumbent. In particular, there exists a threshold s_{subs} such that any platform with $s_1 < s_{\text{subs}}$ delivers matching gains large enough to exceed the incumbent's advertising subsidy, regardless of the rival's technology. A platform that crosses this threshold can attract consumers by offering superior match quality, even against a rival that bundles free access with extensive ancillary services. In this regime, the market rewards technological investment: a large enough innovation disrupts the ad-funded equilibrium.

Corollary 2. *When $B(V) \geq B(x^*)$, there is a threshold s_{subs} such that $B(x(s_{\text{subs}})) = B(x^*)$, and a platform with technology $s_1 < s_{\text{subs}}$ dominates the market against any rival with technology $s_2 > s_1$. When $B(V) < B(x^*)$ instead a platform with $s_2 = \bar{s}$ dominates the market regardless of the technology s_1 of the better rival.*

Equivalently, the condition $B(V) \geq B(x^*)$ can be stated as a condition on primitives: does the advertising revenue at the offline threshold exceed or fall short of the gap $V - x^*$? This provides a testable prediction about which markets are susceptible to technological disruption. In product markets where $V - x^*$ is large, that is, where the gap between the ideal match and the offline reservation value is wide, a sufficiently advanced platform may tip the market onto its product. Intuitively, the model predicts this to be more likely in markets with large product differentiation and moderate offline frictions. Viceversa, product markets suffering from high offline frictions and thick tails are more likely to see entrenched legacy platforms.

Discussion. The analysis above connects two observations about the search industry. First, it offers a rationale for why traditional search intermediaries, despite their technological limitations, have proven remarkably resilient against technological improvements. Modest improvements erode the market power that funds advertising without sufficiently improving user value. In this interpretation, legacy intermediaries don't rely on network effects nor switching costs, but instead rely on the subsidies that the advertising profits allow them to offer to consumers.

Second, the model suggests that disruption, when it occurs, may be discontinuous. A sufficiently advanced technology may tip the market to a state where further innovation becomes profitable, whereas an incrementally improved intermediary may not suffice. AI-powered search assistants may arguably represent such a leap, and consistent with this prediction their revenue model depends on subscriptions and the race to better technologies is on.

5 Conclusions and Limitations

This paper presents a simple model of search intermediation that yields a sharp and economically transparent characterization of the optimal revenue composition for such platforms.

The model embeds a platform design problem into the classical sequential search framework of Wolinsky (1986). I analyze two monetization regimes. Under subscriptions, the platform extracts the incremental consumer surplus generated by its search technology. Under advertising, the platform sells a firm the privilege of being the consumer's first point of contact. The firm then prices as a local monopolist against the consumer's continuation value, and the platform extracts this monopoly rent through an auction. The main result establishes the existence of a unique threshold in the platform's search technology: advertising dominates if the technology falls above the threshold, and subscriptions dominate below it.

On the one hand, the model explains why advertising has been the dominant business model among traditional search intermediaries, which entered markets with pre-existing search frictions and offered incremental improvements. On the other hand, the model predicts the emerging shift toward subscriptions among AI-powered search assistants and similarly advanced search platforms, which facilitate more dramatic reductions in search costs.

The model is deliberately stylized. As such, several modeling choices deserve skepticism. First, the analysis is complete for the limit case of infinitely many firms. The analysis and main result, Theorem 1, can be straightforwardly extended to the finite market case. The only difficulty lies in the observation of Armstrong, Vickers, and Zhou (2009) that pay-for-prominence generically induces asymmetric pricing. In particular, the advertising firm sets a lower price than its competition.

Nonetheless, leveraging the analysis of Armstrong, Vickers, and Zhou (2009) one can show that the same intuitions valid for the limit market hold in the finite case.

Second, the model assumes that firms are symmetric, which leads to a degenerate advertising auction. This is a deliberate choice that helps isolate the key mechanism of the paper, but not an inconsequential one. In particular, with firm heterogeneity the auction would allocate the slot to the firm with the highest willingness to pay, introducing a targeting dimension that interacts with the advertising-subscription tradeoff. Heterogeneous firms represent a natural extension of the current framework, and understanding how targeting changes the threshold is an interesting direction for future work.

Third, and related to the previous point, because firms are ex-ante symmetric, advertising is non-distortionary in this model. In a sense, in this model advertising benefits the consumer by subsidizing improved search technology. Modeling consumer harm from advertising would likely tilt the dominant monetization model towards subscriptions.

Finally, the comparison of pure subscriptions and pure advertising is without loss because the two revenue streams are additive. A platform that charges both a subscription fee and runs an advertising auction earns $\Pi_{\text{hybrid}} = \Pi_{\text{sub}} + \Pi_{\text{ad}}$, and the threshold $\bar{s}$ characterizes the technology at which the dominant source of revenue shifts. In practice, additivity may break down if advertising were to distort prices, if consumer participation were heterogeneous, or if consumers were to self select into subscription and advertising according to some dimension of heterogeneity such as willingness to pay. While all these are reasonable avenues for generalization, I chose to present the baseline additive case because it delivers some crisp insight while preserving the elegance of the Wolinsky (1986) framework.

Despite these limitations, the paper's core message is intuitive, and robust. The optimal way to finance a search intermediary depends on how much it reduces search frictions. Platforms that merely reduce frictions should sell attention, while platforms that eliminate frictions should sell access.

References

ANDERSON, S. P. AND R. RENAULT (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30, 719–735.

ARMSTRONG, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 668–691.

ARMSTRONG, M., J. VICKERS, AND J. ZHOU (2009): "Prominence and Consumer Search," *RAND Journal of Economics*, 40, 209–233.

ATHEY, S. AND G. ELLISON (2011): "Position Auctions with Consumer Search," *Quarterly Journal of Economics*, 126, 1213–1270.

BANCHIO, M., B. LIU, AND A. PERLROTH (2026): "Controlling the Conversation," *Working Paper*.

BANCHIO, M., A. MEHTA, AND A. PERLROTH (2025): "Ads in Conversations," in *Proceedings of the 26th ACM Conference on Economics and Computation*, 350–350.

BERGEMANN, D., A. BONATTI, AND A. SMOLIN (2026): "Menu Pricing of Large Language Models," .

BHAWALKAR, K., A. PSOMAS, AND D. WANG (2025): "Sponsored Questions and How to Auction Them," *arXiv preprint arXiv:2512.03975*.

BRYNJOLFSSON, E., L. CHEN, AND X. GAO (2025): "Gains from product variety: Evidence from a large digital platform," *Information Systems Research*, 36, 2298–2308.

CHEN, Y. AND C. HE (2011): "Paid Placement: Advertising and Search on the Internet," *Economic Journal*, 121, F309–F328.

DIAMOND, P. A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–168.

DUBEY, A., Z. FENG, R. KIDAMBI, A. MEHTA, AND D. WANG (2024): "Auctions with LLM summaries," in *Proceedings of the 30th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, 713–722.

Duetting, P., V. Mirrokni, R. Paes Leme, H. Xu, and S. Zuo (2024): "Mechanism design for large language models," in *Proceedings of the ACM Web Conference 2024*, 144–155.

Edelman, B., M. Ostrovsky, and M. Schwarz (2007): "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, 97, 242–259.

Eliaz, K. and R. Spiegler (2011): "A Simple Model of Search Engine Pricing," *Economic Journal*, 121, F329–F339.

Gehrig, T. (1993): "Intermediation in Search Markets," *Journal of Economics & Management Strategy*, 2, 97–120.

McCall, J. J. (1970): "Economics of Information and Job Search," *Quarterly Journal of Economics*, 84, 113–126.

Rochet, J.-C. and J. Tirole (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1, 990–1029.

——— (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37, 645–667.

Rubinstein, A. and A. Wolinsky (1987): "Middlemen," *The Quarterly Journal of Economics*, 102, 581–593.

Salz, T. (2022): "Intermediation and competition in search markets: An empirical case study," *Journal of Political Economy*, 130, 310–345.

Soumalias, E., M. J. Curry, and S. Seuken (2024): "Truthful aggregation of LLMs with an application to online advertising," *arXiv preprint arXiv:2405.05905*.

Spulber, D. F. (1996): "Market microstructure and intermediation," *Journal of Economic perspectives*, 10, 135–152.

Varian, H. R. (2007): "Position auctions," *International Journal of industrial Organization*, 25, 1163–1178.

White, A. (2013): "Search engines: Left side quality versus right side profits," *International Journal of Industrial Organization*, 31, 690–701.

Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101, 493–511.

A Proofs

Lemma 3 (Existence and Uniqueness of the Reservation Value)**.** *For any* $\sigma \in (0, \mathbb{E}[v])$*, the equation* $\int_x^V (v-x)\,dF(v) = \sigma$ *has a unique solution* $x^* \in (0, V)$*.*

Proof. Define $G(x) = \int_x^V (v-x)\,dF(v)$. We have $G(0) = \mathbb{E}[v] > \sigma$ and $G(V) = 0 < \sigma$. Since $G'(x) = F(x) - 1 < 0$ for all $x \in (0, V)$, the function G is continuous and strictly decreasing on $[0, V]$. By the intermediate value theorem, there exists a unique $x \in (0, V)$ with $G(x) = \sigma$. □

Lemma 4. *Under Assumption 1, the profit function* $\pi_j(p_j) = p_j D_N(p_j, p_N^*)$ *is strictly quasi-concave in* p_j*.*

Proof. For a finite-N market, the demand function $D_N(p_j, p_N^*)$ incorporates both immediate and recall purchases. Anderson and Renault (1999) establishes that this sum is strictly log-concave in p_j when the density $f(\epsilon)$ is strictly log-concave. Then, $\log(\pi_j(p_j)) = \log(p_j) + \log(D_N(p_j, p_N^*))$ is strictly concave. In the large-N limit, this can be more easily proved by observing that demand simplifies to $[1 - F(x^* + p_j - p^*)]/[1 - F(x^*)]$.

□

Proof of Theorem 1

First, I prove the two limit cases discussed in the text.

Part (i). As $s_p \to s$, $x_p^*(s_p) \to x^*$ by continuity. Hence $\Pi_{\text{sub}} = x_p^* - x^* \to 0$, while $\Pi_{\text{ad}} \to p^*[1 - F(x^*)] > 0$.

Part (ii). As $s_p \to 0$, $\int_{x_p^*}^V (v - x_p^*)\,dF(v) \to 0$, which forces $x_p^* \to V$. As $x_p^* \to V$, $\rho(x_p^*) \to \infty$ (the hazard rate is increasing and unbounded on $[0, V)$ under Assumption 1), so $p_p^* = 1/\rho(x_p^*) \to 0$. Then $\Pi_{\text{ad}} = p_p^*[1 - F(x_p^*)] \to 0$ and $\Pi_{\text{sub}} = x_p^* - x^* \to V - x^* > 0$.

The following lemmata prove monotonicity of subscription and advertising revenue, in opposite directions.

Lemma 5 (Monotonicity of Subscription Revenue)**.** $\Pi_{\text{sub}}(s_p)$ *is strictly decreasing in* s_p*.*

Proof. Since $\Pi_{\text{sub}}(s_p) = x_p^*(s_p) - x^*$, we have

$$\frac{d\Pi_{\text{sub}}}{ds_p} = \frac{dx_p^*}{ds_p} = \frac{-1}{1 - F(x_p^*)} < 0,$$

where the second equality follows from the implicit function theorem applied to (1) with s replaced by s_p. □

Lemma 6 (Monotonicity of Advertising Revenue)**.** *$\Pi_{\text{ad}}(s_p)$ is strictly increasing in s_p for all $s_p \in (0, s)$.*

Proof. Writing $\Pi_{\text{ad}}(x_p^*) = [1 - F(x_p^*)]^2 / f(x_p^*)$ and differentiating:

$$\frac{d\Pi_{\text{ad}}}{dx_p^*} = -\frac{[1 - F(x_p^*)]}{[f(x_p^*)]^2}\Big[2[f(x_p^*)]^2 + [1 - F(x_p^*)]f'(x_p^*)\Big].$$

By Assumption 1, $\rho' > 0$ implies $f'(v)[1 - F(v)] + [f(v)]^2 > 0$, so $[1 - F]f' > -f^2$. The bracketed expression exceeds $2f^2 - f^2 = f^2 > 0$. Hence $d\Pi_{\text{ad}}/dx_p^* < 0$, and since $dx_p^*/ds_p < 0$, $d\Pi_{\text{ad}}/ds_p > 0$. □

Finally, to apply the intermediate value theorem, I show a strong form of continuity.

Lemma 7 (Smoothness of Revenue Functions)**.** *Both $\Pi_{\text{sub}}(s_p)$ and $\Pi_{\text{ad}}(s_p)$ are continuously differentiable on $(0, s)$.*

Proof. $x_p^*(s_p)$ is C^1 on $(0, s)$ by the implicit function theorem. The revenue functions are smooth compositions of x_p^*, F, f, and ρ, all of which are smooth on the relevant domain. □

Proof of Proposition 4

(i) Under subscriptions, the binding constraint is $U_p - K = U_{\text{dev}}$ (see Proposition 2), so $U_{\text{sub}} = U_p - K^* = (x_p^* - p_p^*) - (x_p^* - x^*) = x^* - p_p^*$. Since $p_p^* < p^*$, this exceeds $U_{\text{off}} = x^* - p^*$: subscribers retain the price reduction. Under advertising, the consumer pays s_p for her first (directed) search, faces the same acceptance threshold x_p^* and the same price p_p^*. Her expected utility is $U_{\text{ad}} = -s_p + \int_{x_p^*}^{V} (v - p_p^*)\, dF(v) + F(x_p^*)U_p = U_p$

(using $\int_{x_p^*}^{V}(v - x_p^*)\,dF(v) = s_p$). Since $U_p = x_p^* - p_p^* > x^* - p_p^* = U_{\text{sub}}$, consumers strictly prefer advertising. The gap is $U_p - U_{\text{sub}} = x_p^* - x^* = \Pi_{\text{sub}}$.

(ii) Under both regimes, each consumer purchases exactly one unit at price p_p^*. Since marginal cost is zero, aggregate firm revenue (and profit) is p_p^* per consumer. Under subscriptions, this is shared equally: each firm earns p_p^*/N. Under advertising, the winning firm earns $p_p^*[1 - F(x_p^*)]$ from sales but pays $\Pi_{\text{ad}} = p_p^*[1 - F(x_p^*)]$ for the slot, netting zero. The $N-1$ losers earn $F(x_p^*) \cdot p_p^*/(N-1)$ each, totaling $F(x_p^*)p_p^*$. Aggregate firm surplus under advertising is $F(x_p^*)p_p^* = p_p^* - \Pi_{\text{ad}} < p_p^*$.

(iii) Total surplus is consumer surplus + firm surplus + platform surplus. Under subscriptions: $(x^* - p_p^*) + p_p^* + (x_p^* - x^*) = x_p^*$. Under advertising: $U_p + (p_p^* - \Pi_{\text{ad}}) + \Pi_{\text{ad}} = U_p + p_p^* = (x_p^* - p_p^*) + p_p^* = x_p^*$. □

Proof of Lemma 2

Note that $B(x_i) = x_i + \frac{(1-F(x_i))^2}{f(x_i)} = x_i + (1 - F(x_i))H(x_i)$ where $H(x_i)$ is the inverse hazard rate of F. The first and second derivatives of $B(x_i)$ are

$$
\begin{aligned}
B'(x_i) &= F(x_i) + (1 - F(x_i))H'(x_i) \\
B''(x_i) &= f(x_i) + (1 - F(x_i))H''(x_i) - f(x_i)H'(x_i) = \\
&= f(x)(1 + H(x_i)H''(x_i) - H'(x_i))
\end{aligned}
$$

Denoting by $\xi(x_i) = -\log f(x_i)$, we can write $H'(x_i) = H(x_i)\xi'(x_i) - 1$ and $H''(x_i) = H'(x_i)\xi'(x_i) + H(x_i)\xi''(x_i)$. Substituting in the term for $B''(x_i)$, we get

$$
\begin{aligned}
B''(x_i) &= f(x)(1 + H(x_i)H''(x_i) - H'(x_i)) \\
&= f(x)(1 + H(x_i)(H'(x_i)\xi'(x_i) + H(x_i)\xi''(x_i)) - H'(x_i)) \\
&= f(x)\big(1 + H'(x_i)^2 + H(x_i)^2\xi''(x_i)\big)
\end{aligned}
$$

Because $f(x)$ is strictly log-concave, $\xi''(x) > 0$. Since $f(x) > 0$ and $1 + H'(x)^2 \geq 1 > 0$, $B''(x) > f(x) > 0$ for all $x \in (0, V)$, which proves that $B(x)$ is convex.

To prove the second part of the claim, note that the derivative $B'(x_i)$ is monotone increasing over the domain. Since $B'(0) = H'(0) < 0$ (again by log-concavity of the density) and $B'(V) = 1 > 0$, the derivative $B'(x_i)$ is negative over some region $[0, x_{\min})$ and positive over the complement $(x_{\min}, V]$ for some $x_{\min}$ in the interior of the support. That is, $B(x_i)$ is U-shaped. □

Proof of Proposition 5.

Consider the function $C(s)\colon = B(x(s))$. Since $x(s)$ is decreasing and $B(x)$ is convex, $C(s)$ is continuous and quasi-convex. For any s_1 then define

$$g(s_1)\colon = \sup\{s \in [0, \bar{s}]\colon C(s) \leq C(s_1)\}.$$

One can immediately see that $C(s_2) > C(s_1)$ if and only if $s_2 > g(s_1)$.

Continuity of $g(s)$ follows from continuity of the function $C(s)$.

To verify the monotonicity claims made later in the section, it suffices to note that sublevel sets of quasi-convex functions are intervals, and that $C(s)$ remains U-shaped. This immediately implies that $g(s) = s$ for all $s > s_{\min}$ where $s_{\min}$ be such that $x(s_{\min}) = x_{\min}$. Viceversa, for $s < s_{\min}$ the level sets are shrinking as s increases, and hence $g(s)$ is (weakly) decreasing. □

Proof of Corollary 2.

This proof is again immediate from the U-shape of the function $B(x)$ and hence of $C(s)$. If $C(0) < C(\bar{s})$, then the right edge of the sublevel set $\{s\colon C(s) \leq C(0)\}$ is reached at some $s' < \bar{s}$. Clearly, if $C(0) > C(\bar{s})$ then the same sublevel set has $\bar{s}$ as its right edge, and this concludes the proof. □