Dynamic Threats to Credible Auctions[*]

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Abstract

A seller wants to sell a good to a set of bidders using a credible mechanism. We show that when the seller has private information about her cost, it is impossible to implement the optimal mechanism using any static mechanism. In particular, even the optimal first-price auction is no longer credible. We show that the English auction can still credibly implement the optimal mechanism, unlike the optimal Dutch auction. For symmetric mechanisms in which only winners pay, we characterize all the static auctions that are credible: They are first-price auctions that depend only on the seller's cost ex-post via a secret reserve, and may profitably pool bidders via a bid restriction. Our impossibility result highlights the crucial role of *public institutions*, and helps explain the use of *dynamic* mechanisms in informal auctions.

Keywords: Credibility, dynamic deviation, informed principal, mechanism design.

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1 Introduction

In this paper, we revisit the question of credible auction design and show that when the seller has private information about her cost, it is not possible to implement the optimal (i.e., profit-maximizing) mechanism using a static auction. As we show, even the sealed-bid first-price auction is not a credible mechanism. We show that optimality requires a dynamic mechanism and that the ascending auction can be used to credibly implement the optimal mechanism.

We are motivated to study credible auction design because many informal auctions are decentralized, following bilateral communications between the seller and potential buyers. In many informal sales, bidders are contacted with the opportunities to submit bids for a good and do not have a chance to inspect the bids of others or what communication the seller had with other bidders. For example, in bidding for houses, it is often hard for bidders to monitor or audit brokers running auctions regarding bids submitted by others or the communication the brokers have with others. Beyond informal auctions, our study also sheds light on many online auctions where sellers also communicate with bidders through private, bilateral communications.

Our results yield two important takeaways. First, we explain why many informal auction/sale processes are dynamic rather than static: It is impossible for sellers to credibly commit to optimal static mechanisms, and hence under credibility, dynamic mechanisms may overperform static ones. Similarly, this may explain why, in select online markets, sellers implement dynamic mechanisms (for example, eBay uses an auction format similar to the ascending auction). Second, we highlight the role of institutions, such as centralized clearinghouses or public announcements, in ensuring optimal market operations. Simple *public* announcements about offer timing or reserve prices provide the seller with sufficient credibility to restore the optimality of static auctions.

Our notion of credibility follows the approach in Akbarpour and Li (2020). We say that an auction is credible if the only profitable deviations a seller could have are such that they would be observable by some bidder. For example, if a seller attempts to run a second-price auction, it would not be credible because after observing all the bids the seller could charge the winner a price between the highest and second-highest bid (for example, via a practice known as soft reserve prices) and no individual bidder would be able to detect that deviation.

The key new element of our model is that we allow the seller to have private information about her cost and not be able to commit to public messages. For example, the seller of a house may have their own private value of owning the house, a freight broker may have private information about the lowest price they are willing to accept, and an online seller may have a private outside option of selling the item offline.

Our first result is that in this environment the optimal sealed-bid first-price auction is not credible, and in fact has a profitable safe deviation for the seller with probability 1 (Theorem 1). It involves a dynamic deviation by the informed seller. The basic intuition behind why the optimal first-price auction is no longer credible can be understood through the following example (we will explain later why with probability 1 the seller always wants to deviate):

Example 1. Suppose that there are two bidders. Each bidder i has an independent private value $\theta_i \in \{1, 2\}$ (with equal probabilities). The seller has an independent private cost $\theta_0 \in \{0, 0.7\}$. The optimal mechanism can be implemented via a first-price auction as follows. If the seller has cost 0, then she tells each bidder the Myersonian reserve $r(0) = 1$, and solicits a bid from $\{1, \frac{5}{3}\}$; if the seller has cost 0.7, then she tells each bidder i the Myersonian reserve $r(0.7) = 2$, and solicits a bid from $\{0, 2\}$. A bidder of a higher type always selects the higher bid of the two. However, we claim that this mechanism cannot be credible. The deviation works as follows. Consider the seller of cost 0. Let the seller follow the mechanism for bidder 1 and tell bidder 1 that the reserve is $r(0) = 1$. Suppose that the seller gets a high bid $\frac{5}{3}$ from bidder 1. Now, if the seller continues following the mechanism, then regardless of the bid from bidder 2, the seller's payoff will be $\frac{5}{3}$. However, if the seller deviates by pretending to have cost $\hat{\theta}_0 = 0.7$ to bidder 2, then the seller gets a payoff

$$\frac{1}{2} \times \frac{5}{3} + \frac{1}{2} \times 2 > \frac{5}{3},$$

since with probability $\frac{1}{2}$, bidder 2 bids 0 in which case the seller still has the bid $\frac{5}{3}$ from bidder 1, and with probability $\frac{1}{2}$, bidder 2 bids 2, in which case the seller makes strict improvement. □

Of course, in equilibrium, the deviation in Example 1 will not increase the auctioneer's profit because bidder 1 will anticipate the auctioneer using his bid as a reserve for bidder 2 and so reduce his bid to begin with, eventually leading to a lower profit for the auctioneer due to her inability to commit. The reason our result is different than in Akbarpour and Li (2020) is that they assume that the optimal reserve price is commonly known by all bidders, and hence our deviation would be detectable by all bidders. In contrast, since in our model the seller has private information about her cost, such seller deviations are not detectable.

As we show, this result implies that no static mechanism can achieve optimality and be credible at the same time (Theorem 2). The intuition this time follows from Akbarpour and Li (2020) who have shown that optimality plus being static, credible, and winner-paying leaves only one candidate, the first-price auction. Moreover, because the seller has private costs in our setting, it turns out that credibility also requires the mechanism to be winner-paying: In any auction that may require a losing bidder to pay (e.g., all-pay auctions), the seller of a high cost will find it profitable to take the payments from the losing bidders and claim to every bidder that they are outbid by someone else.

Next, we show that the English (ascending) auction can still be used to credibly implement the optimal mechanism (Theorem 3). This result does not require the English auction to be run in an open-cry manner but only through bilateral communications. The intuition for this result is as follows. Consider our deviation for the first-price auction. Suppose that the seller knows that bidder 1 has stayed in the English auction for a while with the clock rising to b_1 (higher than the seller's cost). Now, it might be tempting to conclude that the seller should treat b_1 as the new cost to set the reserve price for bidder 2 (assuming that the seller has not called bidder 2 yet). But that is not optimal. The optimal thing is to ask bidder 2 if he is willing to beat b_1 and then run an auction between him and bidder 1. That is, if the seller could not go back to bidder 1 (like in the first-price auction), then she would inflate the winning bid. But here the seller always wants to go back, and even after such a deviation, she wants to continue as if it were an English auction.

These features of the English auction are not necessarily shared by other dynamic mechanisms, e.g., the Dutch (descending) auction. In Akbarpour and Li (2020), both the optimal English and Dutch auctions are credible. However, in our setting, the optimal Dutch auction may not be credible (Example 2). This is perhaps surprising because in a Dutch auction, our previous deviation for the first-price auction would not work because when bidder 1 sends a message "I will take it at the current price", it is already too late for the seller to use the bid to update the reserve for bidder 2. It turns out that there is a second deviation that the seller may use to profitably manipulate the reserves: Upon observing the bad news that the previous bidders declined to bid given the Myersonian reserve price, reduce the reserve price for the last bidder. The key here is that the last bidder does not know he is the last one — otherwise, the Myersonian reserve price would continue to be optimal — and the seller claims to the last bidder that the competition is high but the bidder is lucky to face a low-cost seller. Note that this downward deviation of the reserve becomes profitable precisely when the seller cannot use the upward deviation of the reserve (i.e., no previous bidders bid above the original reserve) — these two dynamic deviations together also imply that the first-price auction has a profitable safe deviation for the seller with probability 1 (Theorem 1).

Finally, restricting to symmetric mechanisms in which only winners pay, we characterize all the static mechanisms that are credible (Theorem 4). They share three properties. First, they are all first-price in the sense of Akbarpour and Li (2020): When a bidder submits a message to a mechanism, they must know what price they will pay if they win. Second, the message/bid space can be restricted in a way that is independent of the seller's cost (this is a generalization of the ex-ante set price floor). Third, they allow for the seller to incorporate into the mechanism the realization of her cost only in a minimal way via a walk-away option: If the best offer is below the seller's cost, the seller would keep the object (for example, via a practice known as *secrete reserves*).

Among such mechanisms, we show that the seller would always prefer to run an actual auction than use a posted price (Proposition 1). However, as we show, the seller's optimal bid space no longer has a standard interval structure (even under a regular distribution) but may involve substantial bid restrictions (Example 3). The seller's credibility concern leads to a pooling of bidder types that would not appear in the full-commitment solution.

Our model assumes that the auctioneer communicates privately and bilaterally with each of the bidders, following Akbarpour and Li (2020). As we discuss in Section 4.1, another way to escape our impossibility result is to allow for public announcements, where the auctioneer publically announces what the reserve price is after privately observing her cost. This public announcement would make the optimal first-price auction credible. It is a common institution in practice, perhaps precisely to avoid the credibility issue that we highlight. However, in many cases, public announcements may not be feasible, practical, or without costs. Moreover, as we discuss in Section 4.2, even with public announcements, the first-price auction may still suffer a credibility concern where the auctioneer deviates jointly with a bidder before the public announcement. We show that the ascending auction continues to be credible with respect to this type of deviation and satisfies what we call "strong credibility" (see Section 4.2).

1.1 Related Literature

We study credible auction design and uncover a series of dynamic deviations by an informed seller. Our deviations imply an impossibility result for static auctions to achieve both optimality and credibility. We show that the dynamic ascending auction can continue to credibly implement the optimal mechanism. Our model of credible auctions follows Akbarpour and Li (2020) who develop the notion of credibility using an extensive-form game framework.[1] In Akbarpour and Li (2020), the first-price auction, English auction, and Dutch auction are all credible. Among these auctions, we show that only the English auction can continue to credibly implement the optimal mechanism when the seller is privately informed. Unlike in Akbarpour and Li (2020), the deviations we discover involve the manipulation of the timing and reserve prices during the auction. Recently, Komo, Kominers, and Roughgarden (2024) build on the framework of Akbarpour and Li (2020) and Li (2017) to study shill-proofness and characterize the Dutch auction as the unique "strongly shill-proof" auction among the optimal auctions.

We model the privately informed auctioneer following the literature on informed principals (beginning with Myerson 1983; Maskin and Tirole 1990, 1992). In our setting, if the auctioneer has full commitment power, then the privacy of her information about the cost is irrelevant (Myerson 1985; Yilankaya 1999; Skreta 2011; Mylovanov and Tröger 2014). However, we show that under credibility constraints, the seller's private information is no longer irrelevant because it interacts with the potential deviations during the process of the auction. Giovannoni and Hinnosaar (2022) consider a bilateral trade setting where a seller with limited commitment learns about her cost after proposing the contract, and show that the seller can nevertheless obtain the full-commitment payoff.

More broadly, this paper also connects to the literature on auctions and mechanism design with limited commitment (Bester and Strausz 2001; Skreta 2006; Liu, Mierendorff, Shi, and Zhong 2019; Banchio and Yang 2021). Unlike in the Coase-conjecture settings, we model the seller's limited commitment as deviations during the auction rather than after the auction. Thus, unlike Doval and Skreta (2022), we do not have a revelation principle and study instead various forms of dynamic deviations by an informed seller.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 presents the results. Section 4 discusses public announcements and strong credibility. Section 5 concludes.

2 Model

There are one auctioneer, a set of bidders N (with $|N| > 1$), and one good. Each bidder i has a private value $\theta_i \in \Theta_i$, and the auctioneer has a private cost $\theta_0 \in \Theta_0$. We assume that $\Theta_0 = \Theta_i = [0,1]$ for all $i \in N$. The distributions of the bidder values are symmetric and have full support, with a continuously differentiable CDF denoted by F. We assume that the value distribution is regular, i.e., $\theta - \frac{1-F(\theta)}{f(\theta)}$ is strictly increasing. We also assume that

[1] The notion of credibility also appears in Dequiedt and Martimort (2015), though they impose the restriction to revelation mechanisms (e.g., the auctioneer cannot communicate sequentially with bidders).

the distribution of the seller's costs has full support.

A ***mechanism*** is a map that specifies the extensive-form game the auctioneer would run depending on her private information θ_0:

$$M : \Theta_0 \to \mathcal{G}$$

where $\mathcal{G}$ is the set of all finite-depth extensive-form games for the bidders.

Given a mechanism, let $\mathcal{I}_i(\theta_0)$ be the information sets available to bidder $i \in N$, when the auctioneer is of type θ_0. Let

$$\mathcal{I}_i := \bigcup_{\theta_0 \in \Theta_0} \mathcal{I}_i(\theta_0)$$

be the union of all information sets of bidder i over the auctioneer's type θ_0.

An ***interim strategy*** $\sigma_i : \mathcal{I}_i \to A$ for bidder i specifies an action $a \in A$ for every bidder i's information set. An ***ex ante strategy*** $S_i : \Theta_i \to \Sigma_i$ for bidder i specifies an interim strategy $\sigma_i \in \Sigma_i$ for for every type θ_i. A ***protocol*** (M, S_N), where $S_N := (S_i(\cdot))_{i \in N}$, is a pair of a mechanism M and a strategy profile S_N. A protocol (M, S_N) is ***Bayes incentive-compatible (BIC)*** if for all $\theta_i \in \Theta_i$

$$S_i(\theta_i) \in \operatorname*{argmax}_{\sigma_i} \mathbb{E}_{\theta_{-i}, \theta_0} \left[u_i^{M(\theta_0)}(\sigma_i, S_{-i}(\theta_{-i}), \theta_i) \right],$$

where we define

$$u_i^G(\sigma_i, \sigma_{-i}, \theta_i) := u_i(x^G(\sigma_i, \sigma_{-i}), \theta_i)$$

and $u_i(x, \theta_i)$ denotes the utility of agent i with type θ_i given outcome x, and $x^G(\sigma_i, \sigma_{-i})$ denotes the outcome in game G when agents play according to (σ_i, σ_{-i}). A protocol (M, S_N) satisfies ***voluntary participation*** if for all i, there exists σ_i' that ensures bidder i does not receive the good and receives a zero net transfer, regardless of σ_{-i}'. Throughout the paper, we restrict attention to BIC protocols that satisfy voluntary participation.

Given a protocol (M, S_N), a ***messaging game*** is generated as follows: The auctioneer can either choose an outcome or choose to send a message $I_i \in \mathcal{I}_i$ to an agent $i \in N$. Agent i privately observes message I_i and chooses reply $r \in A(I_i)$. The auctioneer privately observes the reply r and repeats. We say that the auctioneer ***plays by the book*** if for every θ_0, *(i)* the auctioneer of type θ_0 selects game $M(\theta_0)$, and then *(ii)* the auctioneer contacts players according to the prescription given by $M(\theta_0)$.

Let $S_0 : \Theta_0 \to \Sigma_0$ denote the auctioneer's ex-ante strategy and $\sigma_0 \in \Sigma_0$ denote the auctioneer's interim strategy, when playing the above messaging game.

Given a promised strategy profile (S_0, S_N), an auctioneer's deviation strategy $\hat{S}_0$ is ***safe*** if, for all agents $i \in N$, and for all type profiles (θ_0, θ_N), there exists a pair $(\hat{\theta}_0, \hat{\theta}_{-i})$ such that

$$o_i\Big(\hat{S}_0, S_N, \theta_0, \theta_N\Big) = o_i\Big(S_0, S_N, \hat{\theta}_0, (\theta_i, \hat{\theta}_{-i})\Big),$$

where o_i denotes the ***observation*** of bidder i given a strategy profile and a type profile. In particular, we assume that each bidder observes whether he wins the object and his own payment.

A safe deviation $\hat{S}_0$ is ***profitable*** for autioneer type θ_0 if

$$\mathbb{E}_{\theta_N}\Big[u_0(S_0(\theta_0), S_N, \theta_0, \theta_N)\Big] < \mathbb{E}_{\theta_N}\Big[u_0(\hat{S}_0(\theta_0), S_N, \theta_0, \theta_N)\Big],$$

where u_0 denotes the auctioneer's utility function given a strategy profile and a type profile. We assume that the auctioneer maximizes the expected profit net of cost. A protocol (M, S_N) is ***credible*** if there is no profitable safe deviation: for any safe deviation $\hat{S}_0$, and any autioneer type θ_0, we have

$$\mathbb{E}_{\theta_N}\Big[u_0(S_0(\theta_0), S_N, \theta_0, \theta_N)\Big] \geq \mathbb{E}_{\theta_N}\Big[u_0(\hat{S}_0(\theta_0), S_N, \theta_0, \theta_N)\Big].$$

An ***outcome*** in our setting is a winner (if any) and a profile of payments $(y, t_N) \in (N \cup \{0\}) \times \mathbb{R}^N_{\geq 0}$. We assume that the transfers are nonnegative, i.e., the auctioneer does not pay the bidders. We allow for randomized mechanisms, in which case the outcome space is given by $\Delta\big((N \cup \{0\}) \times \mathbb{R}^N_{\geq 0}\big)$, and the realization is privately observed by the auctioneer.[2] Given a protocol (M, S_N), we denote its induced allocation rule and transfer rule as $\big(\tilde{y}^{M,S_N}(\cdot), \tilde{t}^{M,S_N}(\cdot)\big) : \Theta_0 \times \Theta_N \to \Delta\big((N \cup \{0\}) \times \mathbb{R}^N_{\geq 0}\big)$. We suppress the dependency on (M, S_N) and the randomization whenever it is clear.

Let

$$\pi(M, S_N, \theta_0) = \mathbb{E}_{\theta_N}\left[\sum_{i \in N} \tilde{t}_i^{M,S_N}(\theta_0, \theta_N) - \mathbb{1}_{\tilde{y}^{M,S_N}(\theta_0,\theta_N) \neq 0} \cdot \theta_0\right]$$

denote the auctioneer's expected profit of protocol (M, S_N).

A protocol (M, S_N) is ***optimal*** if for any BIC protocol $(\widehat{M}, \widehat{S_N})$ that satisfies voluntary participation, we have

$$\mathbb{E}_{\theta_0}\Big[\pi(M, S_N, \theta_0)\Big] \geq \mathbb{E}_{\theta_0}\Big[\pi(\widehat{M}, \widehat{S}_N, \theta_0)\Big].$$

[2] A safe deviation is then defined as one where the auctioneer can provide an innocent explanation by misreporting jointly on $(\theta_0, \theta_{-i}, \varepsilon)$, where ε is the realization of the randomization, to bidder i that keeps bidder i's observation the same.

A protocol (M, S_N) is ***static*** if, for each θ_0, each bidder i has exactly one information set and is called exactly once before any terminal history.

A protocol (M, S_N) is a ***first-price auction*** if (M, S_N) is static, and for each θ_0, each bidder i chooses a bid b_i from a set $B_i(\theta_0) \subset \mathbb{R}_{\geq 0}$ or declines to bid, such that

1. Each bidder i pays b_i if he wins and 0 if he loses
2. If any bidder places a bid, then some maximal bidder wins the object. Otherwise, no bidder wins.

Remark 1. Under our definitions, if the auctioneer were not to be constrained by credibility, then the privacy of her cost would be irreverent. Indeed, an optimal mechanism in our setting simply maps each cost type θ_0 to a corresponding Myersonian optimal auction (e.g., a first-price auction with the cost-dependent Myersonian reserve). Similarly, if the auctioneer did not have private information, our model would collapse to the one studied in Akbarpour and Li (2020).

3 Main Results

In this section, we present our main results. Section 3.1 shows that, once the seller has private costs, the optimal first-price auctions are not credible, and in fact no optimal static mechanisms are credible. Section 3.2 relaxes the requirement of static mechanisms, and shows the ascending auction continues to credibly implement the optimal mechanism. Section 3.3 relaxes the requirement of optimal mechanisms, and characterizes the space of all symetric credible static mechanisms.

3.1 Optimal Static mechanisms Are Not Credible

Our first result shows that, contrary to the case where the auctioneer's cost is publically observable, the optimal first-price auction is no longer credible. In fact, we will show a stronger result: The seller has a profitable safe deviation with probability 1 — the seller would always want to cheat regardless of the cost type and the bidders' types.

Theorem 1. *In any optimal, first-price auction, the seller has a profitable safe deviation with probability* 1*.*

Proof of Theorem 1. Suppose that (M, S_N) is an optimal, first-price auction. By optimality, since the value distributions are symmetric and regular, for almost all θ_0, the induced allocation must be symmetric and deterministic almost everywhere. Then, for any bidder

i, the interim expected payment is pinned down almost everywhere. Thus, the induced bid by bidder i of type θ_i when the auctioneer is of type θ_0

$$b_i(\theta_i;\theta_0)$$

is pinned down up to a measure-zero set. In particular, $b_i(\theta_i;\theta_0)$ can be assumed to be symmetric across bidders, denoted by $b(\theta_i;\theta_0)$.

Moreover, because the bidder value distributions are symmetric and regular, there exists a continuous, strictly increasing function $r(\theta_0)$ such that $b(\theta_i;\theta_0) = 0$ if $\theta_i < r(\theta_0)$ and strictly increasing in θ_i if $\theta_i \geq r(\theta_0)$.[3] Note also that we have

$$b(\theta_i;\hat{\theta}_0) > b(\theta_i;\theta_0)$$

for all $\hat{\theta}_0 > \theta_0$ and $\theta_i > r(\hat{\theta}_0)$.[4]

Let b^* be the maximal bid given by the first $|N|-1$ bidder. With probability 1, we have either $b^* > r(\theta_0)$ or $b^* < r(\theta_0)$.

Case (A). Consider the case of $b^* > r(\theta_0)$. Note also that $b^* < 1 = r(1)$. Now, fix type θ_0 and consider type θ_0's deviation of pretending to be $\hat{\theta}_0$ such that

$$r(\hat{\theta}_0) = b^*$$

and ask the last bidder to bid in the set $B_{|N|}(\hat{\theta}_0)$. The existence of $\hat{\theta}_0$ is guaranteed by continuity of $r(\cdot)$. We claim that this must be a strict improvement for the auctioneer.

There are two cases. First, consider case *(i)* where the last bidder has a value $\theta_{|N|}$ that is weakly below b^*. Then, the auctioneer would get revenue b^* under this deviation. But, even if the auctioneer plays by the book, she also gets the same revenue b^*, since

$$b(\theta_{|N|};\theta_0) \leq \theta_{|N|} \leq b^*,$$

which means the maximal bid the auctioneer gets when playing by the book is b^*.

Now, consider case *(ii)* where the last bidder has a value $\theta_{|N|}$ that is strictly above b^*.

[3]Indeed, $r(\theta_0)$ is pinned down by $r - \frac{1-F(r)}{f(r)} = \theta_0$, which has a unique solution since F is regular.

[4]To see this, note that by the Envelope theorem,

$$b(\theta_i;\theta_0) = \theta_i - \frac{1}{Q(\theta_i;\theta_0)} \int_{r(\theta_0)}^{\theta_i} Q(s;\theta_0)\,\mathrm{d}s$$

where $Q(s;\theta_0)$ is the interim allocation probability. Under an optimal mechanism, we have that $Q(s;\theta_0) = F(s)^{|N|-1}$ for all $s \geq r(\theta_0)$ and hence the claim follows by noting that $r(\theta_0)$ is strictly increasing in θ_0.

Then, we must have that

$$\theta_{|N|} > \max_{i \neq |N|} \theta_i \geq r(\hat{\theta}_0).$$

But then the last bidder would bid even more under the auctioneer's deviation since

$$b(\theta_{|N|}; \hat{\theta}_0) > b(\theta_{|N|}; \theta_0).$$

Since case *(ii)* happens with a positive probability, the safe deviation is a strict improvement for the auctioneer.

Case (B). Consider the case of $b^* < r(\theta_0)$. Fix any type θ_0. We claim that there exists some $\varepsilon > 0$ small enough that the auctioneer of type θ_0 has a strictly profitable deviation by mimicking $\hat{\theta}_0 = \theta_0 - \varepsilon$ to the last bidder.

Since the reserve price $r(\cdot)$ is a strictly increasing, continuous function of the cost type, it suffices to show that type θ_0 wants to slightly decrease the reserve price to the last bidder. Let $r^* := r(\theta_0)$ by the Myersonian reserve for type θ_0. Note that

$$r^* \in \operatorname*{argmax}_r \int_0^1 \mathbb{1}_{\theta_i \geq r}(r - \theta_0) f(\theta_i)\, \mathrm{d}\theta_i$$

which by regularity of F implies that

$$-(r^* - \theta_0) f(r^*) + (1 - F(r^*)) = 0.$$

Note that, under the deviation, when the auctioneer decides what reserve to charge to the last bidder, the above is not the auctioneer's objective function because the last bidder would assume there are still $|N|$ bidders in total. Instead, the objective is

$$W(r) := \int_0^1 \mathbb{1}_{\theta_i \geq r}\Big(b^N(\theta_i; r) - \theta_0\Big) f(\theta_i)\, \mathrm{d}\theta_i,$$

where the last bidder bids according to $b^N(\cdot; r)$ that assumes there are $|N|$ bidders. The bidding function, by the Envelope theorem, is given by

$$b^N(\theta_i; r) = \theta_i - \frac{1}{Q_i(\theta_i; r)} \int_r^{\theta_i} Q_i(s; r)\, \mathrm{d}s$$

for all $\theta_i \geq r$, where we have $Q_i(\theta_i; r) = F(\theta_i)^{|N|-1}$ given the optimality of (M, S_N). Thus,

$$W(r) = \int_0^1 \mathbb{1}_{\theta_i \geq r}\Big(\theta_i - \frac{1}{F(\theta_i)^{|N|-1}} \int_r^{\theta_i} F(s)^{|N|-1}\, \mathrm{d}s - \theta_0\Big) f(\theta_i)\, \mathrm{d}\theta_i.$$

Therefore,

$$W'(r) = -(r-\theta_0)f(r) + \int_r^1 f(\theta_i)\underbrace{\left(\frac{F(r)}{F(\theta_i)}\right)^{|N|-1}}_{<1} \mathrm{d}\theta_i < -(r-\theta_0)f(r) + (1-F(r)).$$

Thus,

$$W'(r^*) < -(r^*-\theta_0)f(r^*) + (1-F(r^*)) = 0.$$

Thus, the type θ_0 auctioneer has a profitable deviation by slightly decreasing the reserve to the last bidder (by mimicking a slightly lower $\hat{\theta}_0$). □

Remark 2. In the proof of Theorem 1, the profitable deviation by the seller is dynamic in the sense that it depends on the information revealed during the process of the auction, even though to each bidder, the deviation is *indistinguishable to a static auction*. This can happen precisely because we allow the seller to have private information, which creates uncertainty over the possible reserves the bidders may face even in a static mechanism.

The first-price auctions are a particular class of static mechanisms. However, our second result, building on Theorem 1 and the results of Akbarpour and Li (2020), shows a completely general impossibility result: There is in fact no static mechanism that is optimal and credible, once the seller has private information.

Theorem 2. *There exists* no *mechanism that is*

(i) credible,

(ii) static, and

(iii) optimal.

Proof of Theorem 2. Suppose for contradiction that (M, S_N) is an optimal, credible, and static mechanism. As in the proof of Theorem 1, by optimality, since the value distributions are symmetric and regular, the induced allocation rule must be deterministic almost everywhere. Moreover, by credibility, the induced transfer rule $\tilde{t}(\theta_N, \theta_0)$ must also be deterministic almost everywhere, by the argument for the proof of Theorem 1 in Akbarpour and Li (2020).[5]

[5]Specifically, we can apply the same argument, treating the realization of the randomization device ε as an opponent type profile θ_{-i}. Intuitively, randomized transfers cannot arise under credibility because the seller has a profitable safe deviation of always asking for the highest possible payment in the support of the randomization.

Step 1. We first show a contradiction under the additional assumption that the mechanism (M, S_N) is also ***winner-paying***, i.e., if for almost all θ_N and θ_0, if $\tilde{t}(\theta_N, \theta_0) \neq 0$, then $\tilde{y}(\theta_N, \theta_0) = i$. Note that by optimality and winner-paying, for almost all θ_0, the induced allocations and transfers $(\tilde{y}(\cdot;\theta_0), \tilde{t}(\cdot;\theta_0))$ must coincide with the Myersonian allocation and transfers almost everywhere. Moreover, note that, for almost all θ_0, $\Big(M(\theta_0), S_N\mid_{\mathcal{I}_N(\theta_0)}\Big)$ must be credible, static, and winner-paying. By Theorem 2 in Akbarpour and Li (2020), for almost all θ_0, $\Big(M(\theta_0), S_N\mid_{\mathcal{I}_N(\theta_0)}\Big)$ must be a first-price auction almost everywhere. Thus, $\Big(M, S_N\Big)$ must be a first-price auction almost everywhere. But Theorem 1 implies that $\Big(M, S_N\Big)$ cannot be credible. A contradiction.

Step 2. We now prove the result without the assumption of winner-paying. In particular, we claim that any optimal, credible, and static mechanism must be winner-paying in our setting. This involves two sub-steps.

Step 2(i). We first claim the auctioneer's payoff for each cost type θ_0 cannot exceed $1-\theta_0$. Note that credibility implies that the auctioneer of type θ_0 has no incentive to mimic another type $\hat{\theta}_0$ when selecting which game G to run. By the Envelope theorem, this implies that

$$U(\theta_0) = \int_{\theta_0}^{1} Q(s)\,\mathrm{d}s + U(1),$$

where $U(\cdot)$ is the equilibrium utility of the auctioneer and $Q(\cdot)$ is the probability of trade, which is pinned down by optimality. Therefore, the auctioneer's expected payoff is

$$\mathbb{E}\Big[U(\theta_0)\Big] = \mathbb{E}\Big[\int_{\theta_0}^{1} Q(s)\,\mathrm{d}s\Big] + U(1).$$

But under any optimal, first-price auction, the auctioneer's expected payoff is $\mathbb{E}\Big[\int_{\theta_0}^{1} Q(s)\,\mathrm{d}s\Big]$, and hence by optimality, we must have $U(1) = 0$. This immediately means that

$$U(\theta_0) = \int_{\theta_0}^{1} Q(s)\,\mathrm{d}s \leq 1 - \theta_0.$$

Step 2(ii). Now, suppose for contradiction that an optimal, credible, and static mechanism (M, S_N) is not winner-paying. Then there exists some auctioneer type $\hat{\theta}_0$ and a positive-measure of type profile θ_N such that the type-$\hat{\theta}_0$ auctioneer gets strictly positive payment from some losing bidder $j \neq \tilde{y}(\theta_N, \hat{\theta}_0)$. Specifically, define $\mathcal{K} \subset \Theta_N$ as

$$\mathcal{K} := \Big\{\theta_N : \exists j \in N \text{ s.t. } \tilde{y}(\theta_N, \hat{\theta}_0) \neq j \text{ and } \tilde{t}_j(\theta_N, \hat{\theta}_0) > 0\Big\},$$

which has a positive measure. Let $\delta := \mathbb{E}\left[\mathbb{1}_{\theta_N \in \mathcal{K}} \sum_{j \neq \tilde{y}(\theta_N, \hat{\theta}_0)} \tilde{t}_j(\theta_N, \hat{\theta}_0)\right] > 0$ denote the expected payment the auctioneer of type $\hat{\theta}_0$ gets from the losing bidders.

Now, consider any auctioneer type $\theta_0 > 1 - \frac{1}{2}\delta$. By **Step 2(i)**, if playing by the book, then the autioneer of type θ_0 can get at most $1 - (1 - \frac{1}{2}\delta) = \frac{1}{2}\delta$. However, consider the following deviation by type θ_0:

- Run the game $\hat{G} = M(\hat{\theta}_0)$
- If the outcome is such that no losing bidder pays the auctioneer, then tell every bidder i that there exists some other bidder j who played according to type $\hat{\theta}_j = 1$.
- Otherwise, collect the payments from the losing bidders, and tell the winner that there exists some other bidder j who played according to type $\hat{\theta}_j = 1$.

Note that this deviation is safe. Moreover, by the optimality of (M, S_N), when following this deviation, the auctioneer would keep the object almost surely and collect the positive payments from the losing bidders. Thus, the expected payoff from this deviation is at least $\delta > \frac{1}{2}\delta$. So (M, S_N) cannot be credible. A contradiction.

Therefore, if (M, S_N) is optimal, credible and static, then (M, S_N) must be winner-paying. But, by **Step 1**, there is no such mechanism, concluding the proof. □

Remark 3. As the proof of Theorem 2 shows, with private costs, the seller also has the temptation of walking away from the auction to collect payments from the losing bidders — hence, credible auctions must be winner-paying. But we also know from Akbarpour and Li (2020), any such static auctions suffer from the deviations of manipulating the transfers, e.g., misrepresenting the second-highest bid in the second-price auction, unless it is a first-price auction. But we have just shown in Theorem 1 that the optimal first-price auctions are not credible due to the dynamic deviations of manipulating the reserves — hence, an impossibility.

3.2 Credibility of Optimal Dynamic Mechanisms

Given the impossibility result (Theorem 2), to maintain credibility, an informed seller must give away the possibility of either having a static auction or an optimal auction.

Our third result shows that if the auctioneer can use dynamic mechanisms, then she can still obtain the optimal profit using a credible mechanism. In particular, she can do so by running an ascending (English) auction.

We say that a protocol (M, S_N) is an ***ascending auction*** if $\left(M(\theta_0), S_N\mid_{\mathcal{I}_N(\theta_0)}\right)$ is an ascending auction as defined in Akbarpour and Li (2020),[6] for each auctioneer type θ_0. In particular, for each θ_0, the bid space of agent i is just Θ_i, which is assumed to be discrete. We also assume the auctioneer's type space is also discrete. The reserve prices are captured by the initial bid for each agent i.

Theorem 3. *There exists an optimal, credible mechanism. In particular, an optimal, ascending auction is credible.*

Proof of Theorem 3. We use a similar argument as in Akbarpour and Li (2020). Note that, in an ascending auction, the misrepresentation of the auctioneer's type and any opponent type would not change the optimal strategy used by bidder i. Specifically, suppose that (M, S_N) is an optimal, ascending auction. Then for each i, S_i specifies the strategy of quitting the auction if and only if the running bid is above agent i's type θ_i. Let S_0' be any safe deviation. Note that the bidder's strategy S_i is also a best response to (S_0', S_{-i}).[7]

Now suppose for contradiction that there exists a safe deviation S_0' that is profitable for some auctioneer type θ_0^*. Then consider the auctioneer's strategy $\hat{S}_0$ defined by playing according to $S_0'(\theta_0^*)$ if the auctioneer's type is θ_0^*, and playing according to $S_0(\theta_0)$ otherwise. Clearly, $\hat{S}_0$ is also a safe deviation. Then the strategy profile $(\hat{S}_0, S_N)$ would induce a BIC mechanism as argued above. But this implies that we have found a BIC mechanism that yields a strictly higher expected profit than (M, S_N) when averaging over seller's types θ_0 (recall we have finite types), contradicting that (M, S_N) is optimal. □

Remark 4. While the proof of Theorem 3 is concise, it is perhaps surprising that our deviation for the first-price auction that involves the manipulation of the reserve prices no longer works for the ascending auctions. To understand the intuition, suppose that we have two bidders. Suppose that the seller knows that bidder 1 has stayed in the ascending auction for a while with the clock rising to a b_1 higher than the seller's cost. Even though it might be tempting to conclude that the seller should treat b_1 as the new cost to set the reserve price for bidder 2 (assuming that the seller has not called bidder 2 yet), it is *not* optimal to do so. This is because the seller can go back to bidder 1 in the ascending auction. The optimal thing is to ask bidder 2 if he is willing to beat b_1 and then run an auction between him and bidder 1. If the seller could not go back to bidder 1, as in the first-price auction, then she would have reason to inflate the winning bid. But here the

[6] See Definition 14 in Akbarpour and Li (2020). For details, see Appendix B.

[7] See Lemma 3 of Akbarpour and Li (2020). In Appendix B, we also formally prove a generalization of their Lemma 3 and use it to show "strong credibility" of the optimal ascending auction (see Section 4.2).

seller always has an incentive to go back, and even after such a deviation, she intends to continue *as if it were an ascending auction*.

As the above discussion suggests, the credibility of the English auction in our setting relies on a few key features of the English auction, which may not be shared by other dynamic mechanisms, e.g., the Dutch (descending) auction. The next example shows that, contrary to the case where the auctioneer's cost is publically observable, the optimal Dutch auction may not be credible:

Example 2 (Optimal Dutch auction is not credible)**.** There are two cost types $c_1 = 0, c_2 = 0.7$. There are two value types $v_1 = 1, v_2 = 2$ with equal probabilities. The Myersonian reserves are $r(c_1) = 1, r(c_2) = 2$. For a fixed set of bidders N, the bidding function is

$$b(v_1; r = 1) = 1, \qquad b(v_2; r = 1) = k_N$$

$$b(v_1; r = 2) = 0, \qquad b(v_2; r = 2) = 2$$

where $k_N \to 2$ as $|N| \to \infty$. Pick some N^* such that $k_{N^*} \geq 1.8$. Suppose that we have $|N^*|$ bidders. Consider the cost type c_2. Consider the event that the first $|N^*| - 1$ bidders stay out of the auction given the reserve 2. Now, consider the safe deviation that, when facing the last bidder, the autioneer pretends to be cost type c_1 and sets reserve price $r = 1$. The last bidder will bid 1 if low type and k_N if high type, which means that the autioneer gets profit

$$1 \times \frac{1}{2} + k_N \times \frac{1}{2} - 0.7 \geq 1.4 - 0.7 = 0.7\,.$$

If playing by the book and setting $r = 2$ for the last bidder, the autioneer only gets $\frac{1}{2} \times (2 - 0.7) = 0.65 < 0.7$. Hence, this deviation is profitable, and the Dutch auction is not credible. □

Remark 5. The deviation in Example 2 is different from our deviation for the first-price auction. Indeed, our deviation for the first-price auction cannot work for the Dutch auction because the clock is running down — when bidder 1 sends a message "I will take it at the current price", it is already too late for the seller to use the bid to update the reserve for bidder 2. Instead, the deviation in Example 2 considers the event where the previous bidders have values lower than the Myersonian reserve, and then reduces the reserve faced by the last bidder. Importantly, the last bidder does not know he is the only one left — otherwise, the seller would optimally set the same reserve price regardless of the number of bidders (Myerson 1981). In equilibrium, once the bidders anticipate this deviation, then they will shade their bids even more, eventually leading to a profit loss for the seller due to her inability to commit.

3.3 Credible Static Mechanisms

Now, we relax the requirement of optimal mechanisms, and study the space of credible and static mechanisms. In practice, static mechanisms (or "sealed-bid auctions") may have other advantages such as saving communication costs. Under the assumption of symmetry and winner-paying, our fourth result characterizes the space of credible static mechanisms, and shows that it however significantly limits the flexibility of the seller.

Since we will be studying suboptimal mechanisms in this section (by Theorem 2), we will be more explicit about randomized mechanisms. In particular, let $\varepsilon \in [0,1]$ be a randomization device and write $\tilde{y}(\theta_N, \theta_0, \varepsilon), \tilde{t}(\theta_N, \theta_0, \varepsilon)$ as the realized allocation and transfer rules.

A protocol (M, S_N) is a ***pay-as-bid*** auction if, for each bidder i, there exists function $b_i(\theta_i, \theta_0)$ such that almost everywhere in $\Theta_N \times \Theta_0 \times [0,1]$, if $\tilde{y}(\theta_N, \theta_0, \varepsilon) = i$, then $\tilde{t}_i(\theta_N, \theta_0, \varepsilon) = b_i(\theta_i, \theta_0)$.

Lemma 1. *Any static, credible mechanism must be a pay-as-bid auction.*

Proof of Lemma 1. Note that for each θ_0, $\left(M(\theta_0), S_N \mid_{\mathcal{I}_N(\theta_0)}\right)$ must also be static and credible. Now, conditional on θ_0, we apply Theorem 1 of Akbarpour and Li (2020) as follows: In particular, for each bidder i and type θ_i, we define $\tilde{\theta}_{-i} := (\theta_{-i}, \varepsilon)$ as an auxiliary "opponent type profile." Since the randomization ε is realized after the auctioneer chooses the game, each bidder i observes no additional information about ε beyond his outcome (whether he wins and his own payment), just like for the opponent type profile θ_{-i}. Therefore, the result follows by applying Theorem 1 of Akbarpour and Li (2020) to the auxiliary opponent type profile, for each auctioneer type θ_0. □

Note that a pay-as-bid auction can still have outcomes that are dependent on the auctioneer types in arbitrary ways. However, we now show that for symmetric and winner-paying mechanisms, the credibility concern due to the seller's own private information will constrain such dependency in a stark way.

A protocol (M, S_N) is ***symmetric*** if, for each θ_0, the induced allocation probability

$$q(\theta_N, \theta_0) := \left(q_i(\theta_N, \theta_0)\right)_{i \in N} := \left(\mathbb{P}\left(\tilde{y}(\theta_N, \theta_0, \varepsilon) = i\right)\right)_{i \in N}$$

is symmetric across the bidders. A protocal (M, S_N) is ***winner-paying*** if for almost all θ_N and θ_0, if $\tilde{t}(\theta_N, \theta_0) \neq 0$, then $\tilde{y}(\theta_N, \theta_0) = i$.

A protocol (M, S_N) is a ***first-price auction with a walk-away option*** if (M, S_N) is static, and for each θ_0, each bidder i chooses a bid b_i from a set $B_i(\theta_0) \subset \mathbb{R}_{\geq 0}$ or declines to bid, such that

1. Each bidder i pays b_i if he wins and 0 if he loses
2. If any bidder places a bid, then either some maximal bidder wins the object or the object is kept by the autioneer. Otherwise, no bidder wins.

A first-price auction with a walk-away option (M, S_N) has a ***public bid space*** $B \subset \mathbb{R}_{\geq 0}$ if there exists $B \subset \mathbb{R}_{\geq 0}$ such that $B_i(\theta_0) = B$ for all bidders i and all auctioneer type θ_0.

Two protocols (M, S_N) and (M', S'_N) are ***outcome-equivalent*** if they induce the same ex-post allocation and transfer rules, almost everywhere in $\Theta_N \times \Theta_0$.

Theorem 4. *Any symmetric, winner-paying, static, and credible* (M, S_N) *is outcome-equivalent to a first-price auction with a walkaway option and a public bid space, in which the seller walks away if and only if the maximal bid is weakly higher than the cost.*

Under symmetry and winner-paying, Theorem 4 shows that, once the seller is constrained by credibility, any static mechanism cannot depend on the seller's cost in any *ex ante* way before communicating with the bidder. Indeed, the bid space is public among all bidders, and the mechanism depends on the cost only via the *ex post* comparison of the seller's cost and the maximal bid.

The proof of Theorem 4 is in the appendix. To illustrate the intuition, we first sketch the key argument for why the bid spaces $B(\theta_0)$ do not depend on θ_0, assuming that the mechanism (M, S_N) is a first-price auction where the bid spaces $B(\theta_0)$ are discrete, and that the seller never walks away.

To show a public bid space, it suffices to show that, regardless of the auctioneer's type θ_0, in the bidding game that type-θ_0 auctioneer runs, the symmetric bidding function $b(\theta_i; \theta_0)$ does not depend on θ_0. Because the bidding function is monotone in θ_i, it then suffices to show that the bid distribution $G(\theta_0)$ does not depend on θ_0.

The key idea is to prove by *induction from the top*. To illustrate, suppose that we have two auctioneer types θ_0 and $\hat{\theta}_0$. Let G and $\hat{G}$ denote the two bid distributions associated with θ_0 and $\hat{\theta}_0$, respectively. Let B and $\hat{B}$ be the supports of G and $\hat{G}$, respectively. We assume that B and $\hat{B}$ are finite, and write $B = \{b^1, \ldots, b^n\}$, where $b^1 > \cdots > b^n$, and $\hat{B} = \{\hat{b}^1, \ldots, \hat{b}^m\}$, where $\hat{b}^1 > \cdots > \hat{b}^m$. We further denote the probability mass of G on b^k by μ^k, and similarly the probability mass of $\hat{G}$ on $\hat{b}^k$ by $\hat{\mu}^k$.

First, we claim that the top bid must be the same, i.e., $b^1 = \hat{b}^1$. Suppose for contradiction that this is not the case, e.g., $\hat{b}^1 > b^1$. Then, note that type-θ_0 auctioneer has a profitable safe deviation: First, elicit the bids from $|N|-1$ bidders, and then mimic type $\hat{\theta}_0$ when facing the last bidder if the highest bid the auctioneer receives from the $|N|-1$ bidders is already b^1. Since this contradicts credibility, we have $b^1 = \hat{b}^1$.

Second, we claim that the probability mass on the top bid must also be the same, i.e., $\mu^1 = \hat{\mu}^1$. Suppose for contradiction that this is not the case, e.g., $\hat{\mu}^1 > \mu^1$. Then, by the previous step, type-θ_0 auctioneer has the following profitable safe deviation: First, elicit the bids from $|N|-1$ bidders, and then mimic type $\hat{\theta}_0$ when facing the last bidder if the highest bid the auctioneer receives from the $|N|-1$ bidders is b^2. Indeed, if playing by the book, type-θ_0 auctioneer has only a probability of μ^1 to increase the highest bid from b^2 to b^1, but following this deviation, the auctioneer is strictly better off in expectation.

Now, we claim that $b^2 = \hat{b}^2$. Indeed, if this is not the case, e.g., if $\hat{b}^2 > b^2$, then by the previous step, type-θ_0 auctioneer has again a profitable safe deviation as before: First, elicit the bids from $|N|-1$ bidders, and then mimic type $\hat{\theta}_0$ when facing the last bidder if the highest bid the auctioneer receives from the $|N|-1$ bidders is b^2. This then implies that we must also have $\mu^2 = \hat{\mu}^2$. Indeed, if this is not the case, e.g., if $\hat{\mu}^2 > \mu^2$, then type-θ_0 auctioneer also has a profitable safe deviation: First, elicit the bids from $|N|-1$ bidders, and then mimic type $\hat{\theta}_0$ when facing the last bidder if the highest bid the auctioneer receives from the $|N|-1$ bidders is b^3.

Then, the induction argument continues and we must have $b^k = \hat{b}^k$ for all the bids in the support, and $\mu^k = \hat{\mu}^k$ for all the probability mass associated with the bids. Thus, the two bid distributions G and $\hat{G}$ must be identical.

Remark 6. The actual proof follows this argument closely, after showing that the mechanism (M, S_N) must be outcome-equivalent to a first-price auction where the seller can walk away. In addition, the actual proof generalizes the above induction argument to *(i)* allow arbitrary bid distributions, which may or may not be discrete, and to *(ii)* incorporate that the auctioneer will walk away when the maximal bid is below her cost.

3.3.1 Maximizing Profits among Credible Static Mechanisms

Under symmetry and winner-paying, Theorem 4 asserts that, if the auctioneer uses static credible auctions, then the only degree of freedom that the auctioneer has is the public bid space B. The walkaway option by the seller can also be thought of as having a secret reserve price.

In particular, Theorem 4 reduces the problem of maximizing profits over this class of mechanisms to be simply designing the public bid space B. The next result shows that using an actual auction with a bid space $|B| > 1$ can always generate more profits than using a posted price ($|B| = 1$):

Proposition 1. *Consider any (M, S_N) that obtains the maximal expected profit among symmetric, winner-paying, static, and credible mechanisms. Then, (M, S_N) must be outcome-equivalent to a first-price auction with a walkaway option and a public bid space B with $|B| > 1$.*

Proof of Proposition 1. By Theorem 4, any such protocol (M, S_N) is equivalent to a first-price auction with a walk-away option and a public bid space B in which the seller walks away if and only if the maximal bid is below the cost.

Suppose we use a posted price mechanism, i.e., $|B| = 1$, in particular, $B = \{b_0\}$ for some b_0. We construct an improvement. Let $\tilde{B} = [b_0, 1]$. There exists some equilibrium given this public bid space. In particular, for any bidder i and type $\theta_i \geq b_0$, the type will participate and submit some bid $b(\theta_i) \geq b_0$. Since the seller has the option to walk away, the expected profit must be increased because realization by realization, the profit is increased:

$$\mathbb{1}_{\max_i \theta_i \geq b_0} \max\left\{b_0 - \theta_0, 0\right\} \leq \mathbb{1}_{\max_i \theta_i \geq b_0} \max\left\{b(\max_i \theta_i) - \theta_0, 0\right\}.$$

Hence, the original mechanism cannot attain the maximal expected profit. Since this holds for any posted price mechanism, the result follows. □

However, which public bid space B is optimal for the auctioneer generally depends on the details of the environment. If the auctioneer never walks away upon observing the bids, then it is easy to see that the optimal bid space B is given by an interval $[R^*, \infty)$, where R^* is the Myersonian reserve with respect to the average cost $\mathbb{E}[\theta_0]$.[8] However, perhaps surprisingly, the next example shows that the seller can benefit from restricting bids in a way to induce some pooling over bidder types and then walking away sometimes:

Example 3. Suppose that we have two bidders and the values $\theta_i \sim U[0, 1]$. Suppose that the seller's cost θ_0 is either 0 or 0.5 with equal probabilities. The Myersonian reserve for the average cost type $\mathbb{E}[\theta_0] = 0.25$ is given by $R^* = (1 + 0.25)/2 = 0.625$. The expected

[8] To see this, note that for any interim allocation rule where $Q(\theta_i; \theta_0) = Q(\theta_i)$, we can write the expected profit as $|N| \cdot \int_0^1 Q(\theta_i)\big(\mathrm{MR}(\theta_i) - \theta_0\big) \mathrm{d}F(\theta_i) \mathrm{d}G(\theta_0) = |N| \cdot \int_0^1 Q(\theta_i)\big(\mathrm{MR}(\theta_i) - \mathbb{E}[\theta_0]\big) \mathrm{d}F(\theta_i)$, where F is the CDF for θ_i, G is the CDF for θ_0, and $\mathrm{MR}(\theta_i)$ is the Myersonian virtual value. The regularity of the value distribution then implies that the optimal interim allocation rule is given by the "assortative matching" rule up to a threshold type defined by $\mathrm{MR}(\theta_i) = \mathbb{E}[\theta_0]$.

profit for the seller using the first-price auction with the public bid space $[0.625, \infty)$ is

$$2 \cdot \int_{0.625}^{1} \Big(\mathrm{MR}(\theta_i) - 0.25\Big) \cdot Q(\theta_i)\, \mathrm{d}\theta_i = 2 \cdot \int_{0.625}^{1} \Big(2\theta_i - 1.25\Big) \cdot \theta_i \, \mathrm{d}\theta_i \approx 0.246.$$

However, consider a first-price auction with a public bid space

$$B = \{0.5\} \cup [0.625, \infty)$$

where the seller of type $\theta_0 = 0$ never walks away and seller of type $\theta_0 = 0.5$ walks away for the bid $b = 0.5$. The equilibrium of this auction is characterized by a threshold type θ^* who is indifferent between bidding 0.5 and bidding 0.625, i.e.,

$$\Big(\theta^* - 0.5\Big) \cdot \frac{1}{2} \cdot \Big(\frac{1}{2} \cdot (\theta^* - 0.5) + 1 \cdot 0.5\Big) = \Big(\theta^* - 0.625\Big) \cdot 1 \cdot \theta^*.$$

Then, we have $\theta^* \approx 0.717$. The expected profit for the seller is then given by

$$2 \cdot \int_{0.717}^{1} \Big(\mathrm{MR}(\theta_i) - 0.25\Big) \cdot \theta_i \, \mathrm{d}\theta_i + 2 \cdot \frac{1}{2} \cdot \int_{0.5}^{0.717} \Big(\mathrm{MR}(\theta_i) - 0\Big) \cdot \Big(\frac{1}{2} \cdot (0.717 - 0.5) + 1 \cdot 0.5\Big) \mathrm{d}\theta_i \approx 0.263,$$

which is strictly higher than the seller's profit under the bid space $[0.625, \infty)$. □

Remark 7. In Example 3, the optimal bid space must be restricted to induce some pooling of bidder types: If the bid space is of the form $[R, \infty)$, where $R \leq 0.5$, then the seller can improve it by using $[0.5, \infty)$ (since $\mathrm{MR}(\theta_i) \leq 0$ for all $\theta_i \leq 0.5$), but then it can be further improved to be $[R^*, \infty)$ where $R^* = 0.625$ as argued above. But that is dominated by using the bid space $\{0.5\} \cup [0.625, \infty)$ which induces a bunching of bidders at the bid 0.5. Intuitively, the seller introduces pooling at a lower bid to allow herself to walk away credibly when her cost is realized to be higher than the maximal virtual value. With full commitment, the seller can always do that using cost-dependent ex ante reserve prices, but as we show credibility eliminates any ex ante dependency.

4 Discussion

4.1 Public Announcements

So far we have assumed that the seller communicates with each of the bidders privately. One of the institutions used by auctioneers in some markets is public communication and such institutions could help in the credibility of auctions as we now discuss.

First, an obvious solution to the problem of credibility of the first-price auction with an informed seller is for the seller to commit to announcing publicly the reserve price, before communicating with any of the bidders. This kind of public auction would make first-price auctions with optimal cost-dependent reserves credible.

Second, even if the seller could not announce publicly the reserve, even just announcing publicly the moment all the bids have to be submitted could make the first-price auction credible. For example, the seller would first publicly announce that they will only accept bids on Sunday at noon (as is the case in some real estate sales). Then, the seller could communicate privately with every bidder the reserve price, without collecting any information about bidder valuations or their willingness to bid above the reserve. Since the seller would not collect responses from bidders before Sunday's noon auction, there would be no safe profitable deviation for the seller to the optimal first-price auction.

Third, public announcements could make many other auction formats credible as well. For example, in the case of the sealed-bid second-price auction, public communication in the form of revealing after the auction all bids and the identities of the bidders could make the second-price auction credible. (This is a common practice in the spectrum auctions run by the FCC in the US and by Industry Canada in Canada — there, while identities of bidders are private during the auction, all bids and bidders are revealed publicly after the auction, allowing bidders to verify that the complex auction rules, such as computing VCG payments and "core adjustment" have been followed.) With such public announcements, unless the auctioneer can "invent" bidders, second-price sealed bid auctions would be credible.

In summary, the institution of public announcements that are often used in practice can go a long way toward helping sellers design credible and revenue-maximizing auctions. That said, public announcements of the kind described above are not always feasible, practical, or without cost. First, the seller may prefer to keep the very existence of an auction secret, and contact only a small number of traders. Second, bidders, especially losing bidders may prefer to have their valuations or even participation in the action secret. Third, the time when the good becomes available for an auction may be random and privately observed by the seller. Especially when combined with the desire to keep the auction private, this could undermine strategies like "all bids are only accepted at noon." Finally, public communication can be costly.

4.2 Strong Credibility of Ascending Auctions

So far we have assumed that the only credibility constraint is that the seller does not have any safe deviation that is profitable. We did not consider other types of deviations. However, there may be other deviations that could create problems for running an auction mechanism credibly.

One such deviation is a possibility that the seller could approach one of the bidders and offer them a secret deviation to a different mechanism that is ex-ante beneficial to both parties. Despite that not being a safe deviation, we may nevertheless be concerned about the credibility of such an auction.[9] Such deviations may be tempting to the seller even in the presence of public announcements: while public announcements can be verified, it may be impossible for bidders to verify that the seller has not contacted any other bidders ahead of time.

Formally, an auctioneer-bidder joint deviation strategy $(\hat{S}_0, \hat{S}_j)$ is ***safe*** if, for all agents $i \in N$, $i \neq j$, and for all type profiles (θ_0, θ_N), there exists a pair $(\hat{\theta}_0, \hat{\theta}_{-i})$ such that

$$o_i\Big(\hat{S}_0, \hat{S}_j, S_{-j}, \theta_0, \theta_N\Big) = o_i\Big(S_0, S_N, \hat{\theta}_0, (\theta_i, \hat{\theta}_{-i})\Big).$$

A joint deviation $(\hat{S}_0, \hat{S}_j)$ is ***mutually beneficial*** for auctioneer type θ_0^* and bidder j if:

$$\mathbb{E}_{\theta_N}\Big[u_0(S_0(\theta_0^*), S_N, \theta_0^*, \theta_N)\Big] < \mathbb{E}_{\theta_N}\Big[u_0(\hat{S}_0(\theta_0^*), \hat{S}_j, S_{-j}, \theta_0^*, \theta_N)\Big],$$

and for all θ_j, playing according to $\hat{S}_j(\theta_j)$ is incentive compatible given $\Big(\hat{S}_0(\theta_0^*), S_{-j}\Big)$ and:

$$\mathbb{E}_{\theta_{-j}}\Big[u_j(S_0(\theta_0^*), S_N, \theta_N)\Big] \leq \mathbb{E}_{\theta_{-j}}\Big[u_j(\hat{S}_0(\theta_0^*), \hat{S}_j, S_{-j}, \theta_N)\Big].$$

We say that a protocol (M, S_N) is ***strongly credible*** if it is credible and for every bidder j and seller type, there is no mutually beneficial safe joint deviation.

We argue that the optimal English auction (with reserve prices that depend on the realized seller cost) is strongly credible. However, the first-price auction may not be strongly credible even in the presence of public announcements.

For the first claim, note that as we argued before, the optimal strategies of bidders do not depend on the seller's announcement about reserve prices (unless their value happens

[9] One defense against such un-safe deviations is that even if they are beneficial to the bidder in this auction, they can be detrimental to that bidder in the future or more generally undermine the credibility of that seller in the eyes of that buyer, leading to long-term losses. However, the analysis of credibility of auctions in repeated environments is beyond the scope of this paper.

to be below the reserve price). Moreover, even if the seller learns something about the valuations of a subset of buyers, the optimal continuation mechanism is to revert to the English auction. See Appendix B for details.

For the second claim (that even with public announcements first-price auctions are not strongly credible), return to the example from the introduction (Example 1). The seller with a public announcement would implement the optimal first-price auction in the following way: when they draw a cost of 0, they would announce that the bidders can bid either 1 or $\frac{5}{3}$. When they draw a cost of 0.7, they would announce that only a bid of 2 is allowed. If bidders believe that the seller will follow this mechanism, they would bid $b(1) = 1$ and $b(2) = 5/3$ when the reserve is low and $b(2) = 2$ when the reserve is high. However, even with public announcement, this mechanism is not strongly credible:

Example 4 (FPA with public announcements is not strongly credible)**.** Consider the setting of Example 1 and the public announcements as above. Consider the following deviation. Before the public announcement of the reserve, if the seller draws a low θ_0, the seller would approach bidder 1 and make them the following offer: You can bid either 1 or 1.48. If you bid 1, I will announce publicly the reserve price of 1. If you bid 1.48, I will announce publicly a reserve price of 2. If the other bidder beats you, you lose. However, if you bid 1.48 and the other bidder does not meet the reserve price of 2, I will secretly sell you the good at 1.48 (in a private post-auction sale). This is clearly not a safe deviation, but we argue that it is mutually beneficial for the buyer 1 and the seller. First, suppose that the value of buyer 1 is 2. In the original mechanism that buyer expects a profit of $(2 - 5/3) \times 3/4 = 1/4$ (they pay 5/3 when they win and they win 3/4 of the time). In the new mechanism, they get an expected profit of

$$(2 - 1.48) \times 0.5 > 1/4.$$

So that type prefers this secret deviation. Moreover, this joint deviation strategy is incentive compatible for the buyer: by misreporting that their value is 1, the buyer can lower the reserve price to 1, as in the original mechanism, but that yields the buyer payoff 1/4, lower than the payoff from following the proposed strategy.

Second, if the value of the buyer is 1, we return to the original mechanism and the payoffs are the same. So for every type of buyer 1, this is an improvement (and reporting true value is incentive compatible in this case too).

How about the seller? When buyer 1 has a value 2, the expected payoff in the original mechanism is 5/3. In the new mechanism, it is $(1.48 + 2)/2 = 1.74 > 5/3$ (and when the value of buyer 1 is 1, the payoffs are unchanged).[10] □

This example illustrates that even in case some announcements can be made publicly, there are still some deviations (albeit, not fully safe), that could undermine the first-price auction.

5 Conclusion

We study a seller with credibility concerns. We show that when the seller has private information about her cost, it is not possible to implement the optimal mechanism using a static mechanism. As we show, even the optimal first-price auction is no longer credible. We show that optimality requires a dynamic mechanism and that the English auction can be used to credibly implement the optimal mechanism. In contrast, we show that the Dutch auction may not be credible. We characterize all symmetric static auctions that are credible: They are first-price auctions that depend only on the seller's cost ex-post via a secret reserve, and may profitably pool bidders via a bid restriction. Our impossibility result highlights the crucial role of *public institutions*, and helps explain the use of *dynamic* mechanisms in informal auctions.

[10] If a reader is concerned that maybe this asymmetric mechanism is perhaps under commitment better than the symmetric mechanism that we described before, note that the seller benefits only because they keep the deviation secret from bidder 2: if bidder 2 understood this deviation, then when the reserve they face is 1, they would bid $1+\varepsilon$ when their value is 2, not 5/3, and that would reduce the seller expected revenue.

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A Omitted Proofs

A.1 Proof of Theorem 4

Let (M, S_N) be a symmetric, credible, and static mechanism.

Step 1. We first make a sequence of observations about (M, S_N).

First, by Lemma 1, (M, S_N) must be a pay-as-bid auction. Without loss of generality, also define $b_i(\theta_i, \theta_0) = 0$ for any i and any (θ_i, θ_0) such that $\mathbb{P}(y(\theta_i, \theta_{-i}, \theta_0, \varepsilon) = i) = 0$.

Second, note that by assumption, (M, S_N) is also winner-paying.

Third, we claim that, under (M, S_N), the bidding function $b_i(\theta_i, \theta_0)$ must be symmetric, i.e., there exists $b(\theta_i, \theta_0)$ such that $b_i(\theta_i, \theta_0) = b(\theta_i, \theta_0)$ for all i. By symmetry of (M, S_N), the interim allocation probability

$$Q_i(\theta_i, \theta_0) = \mathbb{E}_{\theta_{-i}}[q_i(\theta_i, \theta_{-i}, \theta_0)]$$

must be symmetric across bidders, i.e., there exists some $Q(\theta_i, \theta_0)$ such that $Q_i(\theta_i, \theta_0) = Q(\theta_i, \theta_0)$ for all i. By BIC, for each bidder i, the interim expected payment is pinned down by the interim allocation probability Q_i, almost everywhere in $\Theta_i \times \Theta_0$. But, since (M, S_N) is pay-as-bid and winner-paying, this implies that for all i and j

$$Q_i(\theta_i, \theta_0) b_i(\theta_i, \theta_0) = Q_j(\theta_i, \theta_0) b_j(\theta_i, \theta_0)$$

almost everywhere. Moreover, recall that we set $b_i(\theta_i, \theta_0) = 0$ whenever $Q_i(\theta_i, \theta_0) = 0$. Hence, the bidding function $b_i(\theta_i, \theta_0)$ is symmetric across bidders almost everywhere.

Fourth, we claim that, under (M, S_N), any winner must have a maximal bid that exceeds the seller's private cost, i.e., for all θ_0 and all i, if $y(\theta_N, \theta_0, \varepsilon) = i$, then

$$b(\theta_i, \theta_0) \geq \max\left\{\theta_0, \max_{j \neq i} b(\theta_j, \theta_0)\right\},$$

almost everywhere. Suppose for contradiction that this is not the case. Then there exists some θ_0 and some bidder i such that the set

$$\mathcal{Q} := \left\{(\theta_N, \varepsilon) : y(\theta_N, \theta_0, \varepsilon) = i, b(\theta_i, \theta_0) < \max\left\{\theta_0, \max_{j \neq i} b(\theta_j, \theta_0)\right\}\right\}$$

has a positive measure. But consider the following deviation by the auctioneer of type θ_0:

- Run the game $M(\theta_0)$
- If $y(\theta_N, \theta_0, \varepsilon) = i$ and $b(\theta_i, \theta_0) < \theta_0$, keep the object.

- Otherwise, if $y(\theta_N,\theta_0,\varepsilon)=i$ and $b(\theta_i,\theta_0)<\max_{j\neq i} b(\theta_j,\theta_0)$, allocate to bidder j with the highest bid $b(\theta_j,\theta_0)$, instead of bidder i, and charge bidder j a payment $b(\theta_j,\theta_0)$.

This is clearly a profitable deviation. We argue that this is also safe. By symmetry, as argued in the second observation, for any bidder i and any (θ_i,θ_0), there exist type profile θ'_{-i} and realization ε' such that bidder i loses and pays zero. Moreover, for any bidder j and any (θ_j,θ_0) such that $b(\theta_j,\theta_0)>b(\theta_i,\theta_0)\geq 0$, we have $\mathbb{P}(y(\theta_j,\theta_{-j},\theta_0,\varepsilon)=j)>0$ by construction of b. Hence, there exist some type profile θ'_{-j} and some realization ε' such that bidder j wins and pays $b(\theta_j,\theta_0)$. Thus, the deviation is safe. But then (M,S_N) cannot be credible. A contradiction.

Fifth, we claim that, under (M,S_N), if the maximal bid exceeds the seller's cost, then the seller must allocate the object to some bidder, i.e., if $\max_i b(\theta_i,\theta_0)>\theta_0$, then

$$y(\theta_N,\theta_0,\varepsilon)\neq 0,$$

almost everywhere. Suppose for contradiction that this is not the case. Then there exists some θ_0 such that

$$\mathcal{Q}' := \left\{(\theta_N,\varepsilon) : y(\theta_N,\theta_0,\varepsilon)=0, \max_i b(\theta_i,\theta_0)>\theta_0\right\}$$

has a positive measure. But consider the following deviation by the auctioneer of type θ_0:

- Run the game $M(\theta_0)$
- If $y(\theta_N,\theta_0,\varepsilon)=0$ and $\max_i b(\theta_i,\theta_0)>\theta_0$, allocate the object to bidder i with the highest bid $b(\theta_i,\theta_0)$, and charge bidder i a payment $b(\theta_i,\theta_0)$.

This is clearly a profitable deviation. It is also safe by the same argument in the fourth observation. But then (M,S_N) cannot be credible. A contradiction.

Step 2. Let

$$B := \left\{b(\theta_i,0) : \theta_i\in\Theta_i,\, Q(\theta_i,0)>0\right\}.$$

We show that (M,S_N) must be outcome-equivalent to a first-price auction with a walk-away option and the public bid space B.

The proof of this claim involves three substeps.

Step 2(i). First, for each θ_0, let $G(b;\theta_0)$ denote the CDF of the random variable $b(\theta_i,\theta_0)$. We claim that

$$G(s;\theta_0)=G(s;0)$$

for all $s \in [\theta_0, 1]$. To prove it, define

$$\Phi(s;\theta_0) = \int_s^1 (b-s)\,\mathrm{d}G(b;\theta_0).$$

Fix any $\hat{\theta}_0 \in [0,1]$. Let $G_0(\cdot) := G(\cdot;0)$ and $\hat{G}(\cdot) := G(\cdot;\hat{\theta}_0)$. We first show that

$$\int_s^1 (b-s)\,\mathrm{d}G_0(b) \geq \int_s^1 (b-s)\,\mathrm{d}\hat{G}(b)$$

for a G_0-measure-one set. Suppose for contradiction that this is not the case. Then there exists a G_0-positive-measure set $S \ni s$ such that

$$\int_s^1 (b-s)\,\mathrm{d}G_0(b) < \int_s^1 (b-s)\,\mathrm{d}\hat{G}(b).$$

Now, consider the following deviation by the auctioneer of type 0:

- Run the game $M(0)$
- If the maximal bid of the first $|N|-1$ bidder, $\max_{j<|N|} b(\theta_j;0)$, is in the set S, then give the information set $\mathcal{I}_{|N|}(\hat{\theta}_0)$ to the last bidder.

Since S is a G_0-positive-measure set, by symmetry and independence, the above event is a G_0-positive-measure set. For the last bidder, upon receiving the information set $\mathcal{I}_{|N|}(\hat{\theta}_0)$, his payment-conditional-on-winning is $b(\theta_{|N|}, \hat{\theta}_0)$. For any maximal bid by the first $|N|-1$ bidders s, if playing by the book, by **Step 1**, we know that the auctioneer of type 0 gets

$$s + \int (b-s)\mathbb{1}_{b\geq s}\,\mathrm{d}G_0(b).$$

On the other hand, if following this deviation, by **Step 1**, the auctioneer gets

$$s + \int (b-s)\mathbb{1}_{b\geq s}\,\mathrm{d}\hat{G}(b),$$

which is strictly higher whenever $s \in \mathcal{S}$ which happens with a positive probability. Therefore, type-0 auctioneer has a profitable safe deviation, contradicting the credibility of (M, S_N).

Now, note that

$$\Phi(s;\theta_0) = \int_s^1 (b-s)\,\mathrm{d}G(b;\theta_0) = \int_s^1 (1-G(b;\theta_0))\,\mathrm{d}b$$

is a non-increasing and convex function in s, for any θ_0. By continuity, we have that

$$\Phi(s;0) \geq \Phi(s;\hat{\theta}_0)$$

for all $s \in \text{supp}(G_0)$. We claim that we also have the same inequality for all $s \geq \min\{\text{supp}(G_0)\}$. Suppose for contradiction that there exists some $s \geq \min\{\text{supp}(G_0)\}$ such that

$$\Phi(s;0) < \Phi(s;\hat{\theta}_0).$$

Then $s \notin \text{supp}(G_0)$, and hence s must be in an open interval $(s_1, s_2) \subset ([0,1] \backslash \text{supp}(G_0))$ such that $s_1 \in \text{supp}(G_0)$ and $s_2 \in \text{supp}(G_0)$. In particular, G_0 is constant on the open interval, and hence

$$\Phi(s;0) = \frac{s_2 - s}{s_2 - s_1}\Phi(s_1;0) + \frac{s - s_1}{s_2 - s_1}\Phi(s_2;0)$$

We also know that

$$\Phi(s_1;0) \geq \Phi(s_1;\hat{\theta}_0), \qquad \Phi(s_2;0) \geq \Phi(s_2;\hat{\theta}_0).$$

But then

$$\Phi(s;\hat{\theta}_0) > \Phi(s;0) = \frac{s_2 - s}{s_2 - s_1}\Phi(s_1;0) + \frac{s - s_1}{s_2 - s_1}\Phi(s_2;0) \geq \frac{s_2 - s}{s_2 - s_1}\Phi(s_1;\hat{\theta}_0) + \frac{s - s_1}{s_2 - s_1}\Phi(s_2;\hat{\theta}_0)$$

contradicting to the convexity of $\Phi(\cdot;\hat{\theta}_0)$.

Similarly, we also claim that for all $s \geq \max\{\min\{\text{supp}(\hat{G})\}, \hat{\theta}_0\}$,

$$\Phi(s;\hat{\theta}_0) \geq \Phi(s;0).$$

The proof is exactly symmetric to the above if $\min\{\text{supp}(\hat{G})\} \geq \theta_0$. Suppose $\min\{\text{supp}(\hat{G})\} < \theta_0$. Then, there exists a $\hat{G}$-positive-measure event under which the maximal bid from the first $|N|-1$ bidders is strictly less than $\hat{\theta}_0$. If that happens, the auctioneer of type $\hat{\theta}_0$ can deviate to give the last bidder information set $\mathcal{I}_{|N|}(0)$. In order for this deviation not to be profitable, we must have

$$\Phi(\hat{\theta}_0;\hat{\theta}_0) \geq \Phi(\hat{\theta}_0;0).$$

For all $s > \theta_0$ such that $s \in \text{supp}(G_0)$, the same deviation as before yields the desired inequality. For all $s > \theta_0$ such that $s \notin \text{supp}(G_0)$, the same convexity argument as above would also yield the desired inequality. Therefore, we have

$$\Phi(s;\hat{\theta}_0) \geq \Phi(s;0).$$

for all $s \geq \max\{\min\{\mathrm{supp}(\hat{G})\}, \hat{\theta}_0\}$.

Combining these two sets of inequalities together, we have

$$\Phi(s;\hat{\theta}_0) = \Phi(s;0).$$

for all

$$s \geq \max\Big\{\min\{\mathrm{supp}(G_0)\}, \min\{\mathrm{supp}(\hat{G})\}, \hat{\theta}_0\Big\}.$$

Denote $m_0 = \min\{\mathrm{supp}(G_0)\}, \hat{m} = \min\{\mathrm{supp}(\hat{G})\}$. Consider first the case $\theta_0 < \max\{m_0, \hat{m}\}$. Then, because

$$\Phi(s;\hat{\theta}_0) = \Phi(s;0).$$

for all $s \geq \max\{m_0, \hat{m}\}$, we must have

$$G(s;\hat{\theta}_0) = G(s;0)$$

for all $s \in [0,1]$. Now, suppose $\theta_0 \geq \max\{m_0, \hat{m}\}$, the same argument implies that

$$G(s;\hat{\theta}_0) = G(s;0)$$

for all $s \in [\hat{\theta}_0, 1]$. Since $\hat{\theta}_0$ is arbitrary, this proves the claim.

Step 2(ii). By BIC, $b(\theta_i;\theta_0)$ is non-decreasing in θ_i for all θ_0. By **Step 2(i)**, for any θ_0, we have

$$\max\{b(\theta_i;0), \theta_0\} \overset{d}{=} \max\{b(\theta_i;\theta_0), \theta_0\}$$

Since both $\max\{b(\theta_i;0), \theta_0\}$ and $\max\{b(\theta_i;\theta_0), \theta_0\}$ are non-decreasing in θ_i, the above implies that for any θ_0, we have

$$\max\{b(\theta_i;0), \theta_0\} = \max\{b(\theta_i;\theta_0), \theta_0\}$$

almost everywhere in Θ_i. Now, we claim that the auctioneer can replicate the outcomes by using another protocol $(M', S;_N)$ that is a first-price auction with a walkaway option and a public bid space $B := \Big\{b(\theta_i, 0) : \theta_i \in \Theta_i,\, Q(\theta_i, 0) > 0\Big\}$.

Fix the measure-1 set of types on which

$$\max\{b(\theta_i;0),\theta_0\} = \max\{b(\theta_i;\theta_0),\theta_0\}$$

for all $i \in N$. By **Step 1**, if

$$\max_i\{b(\theta_i;\theta_0)\} > \theta_0$$

then the object must be allocated to a maximal bidder for whom we have $b(\theta_i;0) = b(\theta_i;\theta_0)$. The auctioneer can follow the rules for tie-breaking in (M, S_N).[11] This would result in the same ex-post allocation and the same ex-post payment. By **Step 1**, if

$$\max_i\{b(\theta_i;\theta_0)\} < \theta_0$$

then the object must be kept by the auctioneer, hence resulting in the same allocation and payment. If

$$\max_i\{b(\theta_i;\theta_0)\} = \theta_0$$

then let the auctioneer follow the same rule in (M, S_N) for allocating the object, which would also result in the same allocation and payment.

Now, we claim that the strategy profile $\left\{b(\theta_i;\theta_0)\right\}_{i\in N}$ continues to be BIC. Fix any bidder i. Note that if the auctioneer can announce the game $m(\theta_0) := M(\theta_0)$ as a cheap-talk message but just give the public bid space B for the bidder to choose, then bidder i would have exactly the same information in (M', S'_N) as in (M, S_N). In such a case, for each cheap-talk message, fixing the opponent's strategies $\{b_j(\theta_j;0)\}_{j\neq i}$, bidder i's any strategy would result in the same ex-post outcome as in (M, S_N), almost everywhere. Moreover, bidder i has the same belief about (θ_0, θ_{-i}), and hence following the strategy $b_i(\theta_i;0)$ must be a best-response. But then, since this strategy does not depend on the cheap-talk message $m(\theta_0)$, it must also maximize bidder i's expected payoff even if bidder i does not observe the cheap-talk message $m(\theta_0)$. Hence, (M', S'_N) would also be BIC.

Step 2(iii). Finally, we complete the characterization by noting that the event

$$\max_{i\in N} b(\theta_i;0) = \theta_0$$

can only happen with zero probability, given the independence of θ_0 and θ_N. Therefore,

[11] Such tie-breaking can require cheap talk from the bidders to report their types.

in (M', S'_N), we may let the auctioneer walk away if and only if

$$\max_{i \in N} b(\theta_i; 0) \geq \theta_0,$$

while keeping the resulting outcomes to be equivalent almost everywhere.

B Ascending Auctions

In this appendix, for completeness, we give the formal definition of an ascending auction following Akbarpour and Li (2020). We then show that optimal ascending auctions are strongly credible.

B.1 Definition of Ascending Auctions

As in Akbarpour and Li (2020), we assume the type space is discrete to avoid modeling continuous-time games. Let $\Theta_i := \{\theta_i^1, \dots, \theta_i^K\}$, where $\theta_i^1 = 0$ and $\theta_i^{k+1} - \theta_i^k > 0$, for all $i = 0, 1, \dots, N$ (including the auctioneer's types).

We say that (M, S_N) is an ***ascending auction*** if for every θ_0, $\left(M(\theta_0), S_N \mid_{\mathcal{I}_N(\theta_0)}\right)$ satisfies Definition 14 in Akbarpour and Li (2020).

B.2 Strong Credibility of Ascending Auctions

In this section, we provide a generalization of Lemma 3 in Akbarpour and Li (2020) which will be used to prove the strong credibility of optimal ascending auctions.

Lemma 2. *Let (M, S_N) be an ascending auction. For every bidder i, if $(\hat{S}_0, \hat{S}_j)$ is a safe joint deviation such that $j \neq i$, then S_i is an ex post best response to $(\hat{S}_0, \hat{S}_j, S_{-\{j,i\}})$ for all θ_0 and θ_{-i}.*

Proof of Lemma 2. Let $(\hat{S}_0, \hat{S}_j)$ be a safe joint deviation. Take any type θ_i. We claim that any deviating strategy $\hat{S}_i(\theta_i)$ cannot yield strictly higher payoff for type θ_i.

Suppose that $S_i(\theta_i)$ and deviating strategy $\hat{S}_i(\theta_i)$ choose different actions for the first time after receiving message I_i. There are three cases to consider; we will show that, in each case, $\hat{S}_i(\theta_i)$ is not a profitable deviation for every possible realization of θ_0 and θ_{-i}.

Case 1: Suppose at I_i strategy $\hat{S}_i$ chooses the quit action, thus receiving zero utility. Since $(\hat{S}_0, \hat{S}_j)$ is safe and ascending auction has threshold pricing, the strategy S_i must obtain weakly positive utility by accepting, so the deviation is unprofitable.

Case 2: Suppose at I_i strategy S_i quits while deviation $\hat{S}_i$ accepts. Following the same logic as in the proof of Lemma 3 of Akbarpour and Li (2020), we can find plausible explanations $\hat{\theta}_0, \hat{\theta}_{-i}$ and agent type $\hat{\theta}_i$ such that

$$o_i(\hat{S}_0, \hat{S}_j, \hat{S}_i, S_{-\{j,i\}}, \theta_0, \theta_N) = o_i(S_0, S_N, \hat{\theta}_0, (\hat{\theta}_i, \hat{\theta}_{-i})).$$

If $\tilde{y}(\hat{\theta}_i, \hat{\theta}_0, \hat{\theta}_{-i}) \neq i$, then the deviation is clearly unprofitable by threshold pricing. Now suppose $\tilde{y}(\hat{\theta}_i, \hat{\theta}_0, \hat{\theta}_{-i}) = i$. However, note that $\tilde{y}(\theta_i, \hat{\theta}_0, \hat{\theta}_{-i}) \neq i$ since $S_i(\theta_i)$ can also reach I_i and specifies agent i to quit. But then the misreport $\hat{\theta}_i$ induces the agent to win the object and pay $\tilde{t}_i(\hat{\theta}_i, \hat{\theta}_0, \hat{\theta}_{-i}) > \theta_i$ by threshold pricing and orderly property of the ascending auction, so the deviation is unprofitable.

Case 3: Suppose at I_i the two strategies decide to accept under two different actions. Then, property 5b of the ascending auction guarantees that the two strategies generate the same utility. □

The following result strengthens Theorem 3 to the notion of strong credibility.

Theorem 5. *There exists an optimal, strongly credible auction. In particular, an optimal, ascending auction is strongly credible.*

Proof of Theorem 5. Suppose that (M, S_N) is an optimal, ascending auction. By Theorem 3, we know (M, S_N) is credible. Now, to prove strong credibility, suppose for contradiction that there exists a safe joint deviation $(S', \hat{S}_j)$ that is mutually beneficial between the auctioneer of type θ_0^* and bidder j.

Consider the following strategy by the auctioneer. If the auctioneer's type is $\theta_0 \neq \theta_0^*$, run the optimal ascending auction as in S_0. If the auctioneer's type is θ_0^*, deviate to $S'(\theta_0^*)$ and ask bidder j to play according to $\hat{S}_j$. By the definition of mutually beneficial deviation, $\hat{S}_j$ must be optimal for bidder j to play against $(S'(\theta_0^*), S_{-j})$. Moreover, by Lemma 2, we also know that S_i is a best reply to $(S', \hat{S}_j, S_{-\{j,i\}})$ even after knowing the auctioneer's type. Therefore, we know that

$$\left(S'(\theta_0^*), \hat{S}_j, S_{-j}\right)$$

induces a BIC mechanism (where everyone knows the auctioneer's type is θ_0^*) that yields the auctioneer a payoff strictly higher than what the auctioneer can obtain following $S_0(\theta_0^*)$. But then if the auctioneer follows the above strategy, then the auctioneer can guarantee an expected payoff in a BIC mechanism that is strictly higher than what she could obtain in the optimal, ascending auction (recall we have discrete types), contradicting that the ascending auction is optimal. □